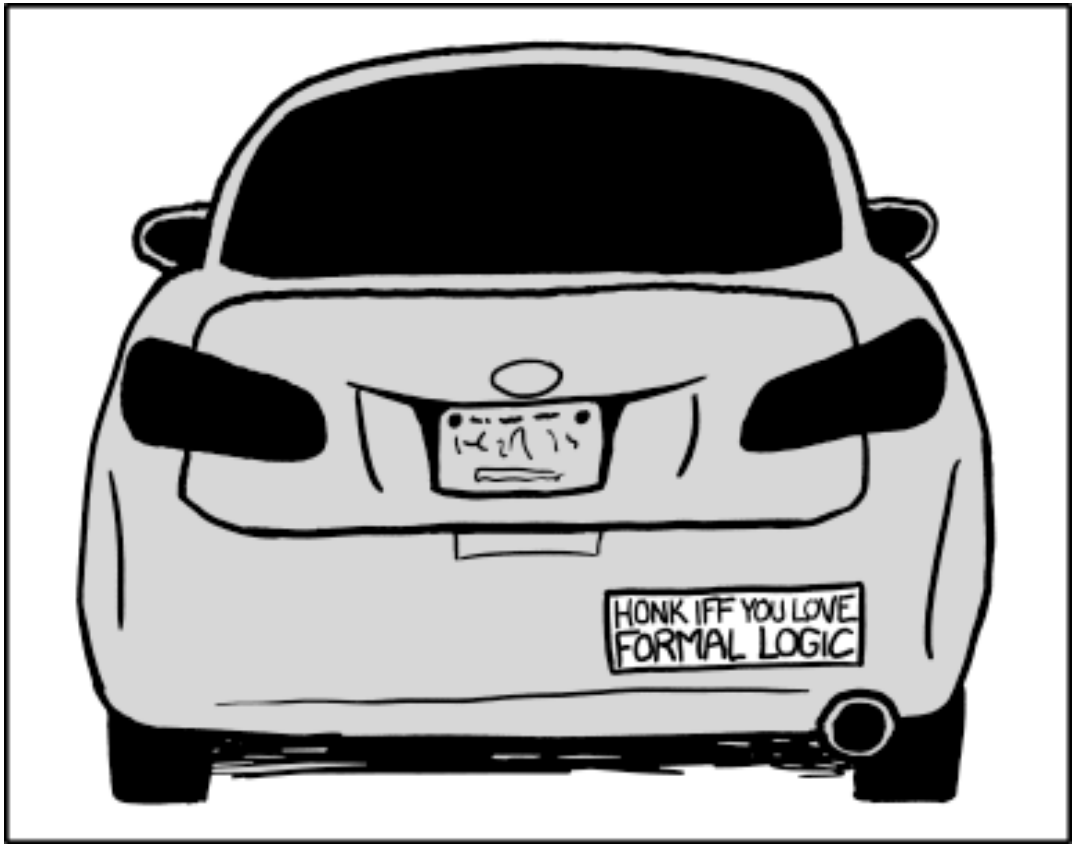
CSE 311: Foundations of Computing I

Lecture 1: Propositional Logic



What is logic and why do we need it?

Logic is a language, like English or Java, with its own

- words and rules for combining words into sentences (syntax)
- ways to assign meaning to words and sentences (semantics)

Why learn another language?

We know English and Java already?

Why not use English?

- Turn right here…

  Does “right” mean the direction or now?

- We saw her duck

  Does “duck” mean the animal or crouch down?

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo

  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

Natural languages can be unclear or imprecise

Why learn a new language?

We need a language of reasoning to

- state sentences more precisely
- state sentences more concisely
- understand sentences more quickly

Formal logic has these properties

Propositions: building blocks of logic

A ***proposition*** is a statement that

- is either true or false
- is "well-formed"

Propositions: building blocks of logic

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All cats are mammals

true

All mammals are cats

false

Are These Propositions?

$2 + 2 = 5$

This is a proposition. It’s okay for propositions to be false.

$x + 2 = 5389$, where x is my PIN number

This is a proposition. We don’t need to know what x is.

Akjsdf!

Not a proposition because it’s gibberish.

Who are you?

This is a question which means it doesn’t have a truth value.

Every positive even integer can be written as the sum of two primes.

This is a proposition. We don’t know if it’s true or false, but we know it’s one of them!

Propositions

We need a way of talking about *arbitrary* ideas...

Propositional Variables: $p, q, r, s, \ldots$

Truth Values:

- **T** for **true**
- **F** for **false**

Familiar from Java

- Java `boolean` represents a truth value
  - constants `true` and `false`
  - variables hold *unknown* values
- Operators that calculate new truth values from given ones
  - unary: not (`!`)
  - binary: and (&&), or (||)

Logical Connectives

Negation (not)	$\neg p$
Conjunction (and)	$p \wedge q$
Disjunction (or)	$p \vee q$
Exclusive Or	$p \oplus q$
Implication	$p \longrightarrow r$
Biconditional	$p \longleftrightarrow r$

Some Truth Tables

p	$\neg p$
T	
F	

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

p	q	$p \oplus q$
T	T	
T	F	
F	T	
F	F	

Some Truth Tables

p	$\neg p$
T	F
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F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Logic forces us to distinguish $\vee$ from $\oplus$

Implication

"If it's raining, then I have my umbrella"

p	r	$p \rightarrow r$
T	T	T
T	F	F
F	T	T
F	F	T

It's useful to think of implications as promises. That is "Did I lie?"

	It's raining	It's not raining
I have my umbrella		
I do not have my umbrella		

Implication

"If it's raining, then I have my umbrella"

p	r	$p \rightarrow r$
T	T	T
T	F	F
F	T	T
F	F	T

It's useful to think of implications as promises. That is "Did I lie?"

	It's raining	It's not raining
I have my umbrella	No	No
I do not have my umbrella	**Yes**	No

The only lie is when:

(a) It's raining AND

(b) I don't have my umbrella

Implication

"If the Seahawks won, then I was at the game."

p	r	$p \rightarrow r$
T	T	T
T	F	F
F	T	T
F	F	T

What's the one scenario where I lied?

	I was at the game	I wasn't at the game
Seahawks won		
Seahawks lost		

Implication

"If the Seahawks won, then I was at the game."

p	r	$p \rightarrow r$
T	T	T
T	F	F
F	T	T
F	F	T

What's the one scenario where I lied?

	I was at the game	I wasn't at the game
Seahawks won	Ok	**I lied**
Seahawks lost	Ok	Ok

Implication

"If it's raining, then I have my umbrella"

p	r	$p \to r$
T	T	T
T	F	F
F	T	T
F	F	T

Are these true?

$2 + 2 = 4 \to$ *earth is a planet*

The fact that these are unrelated doesn't make the statement false! "2 + 2 = 4" is true; "earth is a planet" is true. T→T is true. So, the statement is true.

$2 + 2 = 5 \to$ *26 is prime*

Again, these statements may or may not be related. "2 + 2 = 5" is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!

$p \rightarrow r$

(1) "I have collected all 151 Pokémon if I am a Pokémon master"

(2) "I have collected all 151 Pokémon only if I am a Pokémon master"

$p \to r$

(1) "I have collected all 151 Pokémon if I am a Pokémon master"

(2) "I have collected all 151 Pokémon only if I am a Pokémon master"

These sentences are implications in opposite directions:

(1) "Pokémon masters have all 151 Pokémon"

(2) "People who have 151 Pokémon are Pokémon masters"

So, the implications are:

(1) If I am a Pokémon master, *then* I have collected all 151 Pokémon.

(2) If I have collected all 151 Pokémon, *then* I am a Pokémon master.

$p \to r$

p	r	$p \to r$
T	T	T
T	F	F
F	T	T
F	F	T

Implication:

– p implies r

– whenever p is true r must be true

– if p then r

– r if p

– p only if r

– p is sufficient for r

– *r is necessary for p*

Biconditional: $p \leftrightarrow r$

- *p* if and only if *r*   (*p* iff *r*)
- *p* implies *r* and *r* implies *p*
- *p* is necessary and sufficient for *r*

p	r	$p \leftrightarrow r$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional: $p \leftrightarrow r$

- *p* if and only if *r* (*p* iff *r*)
- *p* implies *r* and *r* implies *p*
- *p* is necessary and sufficient for *r*

p	r	$p \leftrightarrow r$	$p \rightarrow r$	$r \rightarrow p$	$(p \rightarrow r) \wedge (r \rightarrow p)$
T	T	T	T	T	
T	F	F	F	T	
F	T	F	T	F	
F	F	T	T	T	

Biconditional: $p \leftrightarrow r$

- *p* if and only if *r* (*p* iff *r*)
- *p* implies *r* and *r* implies *p*
- *p* is necessary and sufficient for *r*

p	r	$p \leftrightarrow r$	$p \rightarrow r$	$r \rightarrow p$	$(p \rightarrow r) \wedge (r \rightarrow p)$
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	T	F	F
F	F	T	T	T	T

A Compound Proposition

"Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna"

We'd like to *understand* what this proposition means.

A Compound Proposition

"Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna"

We'd like to *understand* what this proposition means.

First find the simplest (**atomic) propositions**:

q "Garfield has black stripes"

r "Garfield is an orange cat"

s "Garfield likes lasagna"

(q if (r and s)) and (r or (not s))

Logical Connectives

Negation (not)	$\neg p$
Conjunction (and)	$p \land q$
Disjunction (or)	$p \lor q$
Exclusive Or	$p \oplus q$
Implication	$p \longrightarrow r$
Biconditional	$p \longleftrightarrow r$

q	"Garfield has black stripes"
r	"Garfield is an orange cat"
s	"Garfield likes lasagna"

"Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna"

$\downarrow$

(q if (r and s)) and (r or (not s))

Logical Connectives

Negation (not)	$\neg p$
Conjunction (and)	$p \wedge q$
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q	"Garfield has black stripes"
r	"Garfield is an orange cat"
s	"Garfield likes lasagna"

"Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna"

$\Downarrow$

(q if (r and s)) and (r or (not s))

$\Downarrow$

$((r \wedge s) \longrightarrow q) \wedge (r \vee \neg s)$

Analyzing the Garfield Sentence with a Truth Table

q	r	r	$((q \land r) \to p) \land (q \lor \neg r)$
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

Analyzing the Garfield Sentence with a Truth Table

q	r	s	$r \vee \neg s$	$(r \wedge s) \to q$	$((r \wedge s) \to q) \wedge (r \vee \neg s)$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Analyzing the Garfield Sentence with a Truth Table

q	r	s	$\neg s$	$r \vee \neg s$	$r \wedge s$	$(r \wedge s) \rightarrow q$	$((r \wedge s) \rightarrow q) \wedge (r \vee \neg s)$
F	F	F					
F	F	T					
F	T	F					
F	T	T					
T	F	F					
T	F	T					
T	T	F					
T	T	T					

Analyzing the Garfield Sentence with a Truth Table

q	r	s	$\neg s$	$r \vee \neg s$	$r \wedge s$	$(r \wedge s) \to q$	$((r \wedge s) \to q) \wedge (r \vee \neg s)$
F	F	F	T	T	F	T	T
F	F	T	F	F	F	T	F
F	T	F	T	T	F	T	T
F	T	T	F	T	T	F	F
T	F	F	T	T	F	T	T
T	F	T	F	F	F	T	F
T	T	F	T	T	F	T	T
T	T	T	F	T	T	T	T