Sets Reference Sheet

Common Sets

- $\mathbb{N} = \{0, 1, 2, \dots\}$ is the set of *Natural Numbers*.
- $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is the set of *Integers*.
- $\mathbb{Q} = \left\{ \frac{p}{q} \ : \ p, q \in \mathbb{Z} \wedge q \neq 0 \right\}$ is the set of *Rational Numbers*.
- $\mathbb{R}$ is the set of *Real Numbers*.

Containment, Equality, and Subsets

Let A, B be sets. Then:

- $x \in A$ ("x is an *element* of A") means that x is an element of A.
- $x \notin A$ ("x is *not* an *element* of A") means that x is *not* an element of A.
- $A \subseteq B$ ("A is a *subset* of B") means that all the elements of A are also in B.
- $A \not\subseteq B$ ("A is a **not** a *subset* of B") means that some element of A is not also in B.
- $A \supseteq B$ ("A is a *superset* of B") means that all the elements of B are also in A.
- $(A = B) \equiv (A \subseteq B) \wedge (B \subseteq A) \equiv \forall x \ (x \in A \leftrightarrow x \in B)$

Set Operations

Let A, B be sets. Then:

- $A \cup B$ is the *union* of A and B. $A \cup B = \{x \ : \ x \in A \vee x \in B\}$.
- $A \cap B$ is the *intersection* of A and B. $A \cap B = \{x \ : \ x \in A \wedge x \in B\}$.
- $A \setminus B$ is the *difference* of A and B. $A \setminus B = \{x \ : \ x \in A \wedge x \notin B\}$.
- $A \oplus B$ is the *symmetric difference* of A and B. $A \oplus B = \{x \ : \ x \in A \oplus x \in B\}$.
- $\overline{A}$ is the *complement* of A. If we restrict ourselves to a "universal set", $\mathcal{U}$, (a set of all possible things we're discussing), then $\overline{A} = \{x \in \mathcal{U} \ : \ x \notin A\}$.

Set Constructions

Let A, B, C, D be sets. Then:

- $S = \{x \ : \ \mathsf{P}(x)\}$ is *set builder notation* which means S is the set that contains all objects x with property P (and no other elements).
- $A \times B$ is the *cartesian product* of A and B. $A \times B = \{(a, b) : a \in A, \ b \in B\}$.
- $[n]$ ("*brackets* n") is the set of integers from 1 to n. $[n] = \{x \in \mathbb{Z} \ : \ 1 \leq x \leq n\}$.
- $\mathcal{P}(A)$ is the *power set* of A. $\mathcal{P}(A) = \{S \ : \ S \subseteq A\}$. $\mathcal{P}(A)$ is the set of all subsets of A.