Consider the following statements.

Suppose each cow is only one color.
Every group of cows contain only cows of the same color.

Spoof by strong induction:
P(1): all groups of cows of size one contain only cows of one color.
(IH): $P(k) \rightarrow P(k + 1)$ for all $k \geq 1$.

Consider a group of $k + 1 \geq 2$ cows.
By our IH, all groups of size $\geq k$ contain cows of only one color.
So, break the $k + 1$ cows into 2 overlapping subgroups;
these by our IH are all the same color,
and since they share a cow in common the 2 groups are the same color.