Homework 4: Common Bugs and Comments

Since there were a few very common mistakes, we wanted to discuss what they were, and how to correct them.

1. Always introduce your variables.

Remember that even if a statement doesn't say "for all" it might implicitly be a for all (like the statement "even integers greater than 10 are not prime")

If you want to prove a for all, you need to introduce an arbitrary variable; for 311, you should always call arbitrary variables "arbitrary" (even if the statement we're proving doesn't call them arbitrary).

2. But only call a variable arbitrary if you mean it to be arbitrary.

If you're showing an existential, your variable is not arbitrary (at least it shouldn't be!).

Just because b is arbitrary does not mean that $b + 1$ is arbitrary (or any modification is arbitrary). Imagine you let b be an arbitrary natural number. $b + 1$ is not arbitrary – it's positive! $b + 1$ cannot be 0, so it's not arbitrary

You should use arbitrary to mean "I can put in any element here, and the rest of this argument will work." And furthermore, you should only use "arbitrary" if you intend to indicate that a for all statement can be derived at the end.

3. When proving an existential (or disproving a forall statement) give a very particular example.

Trying to give a general form (e.g. saying "choose any B, C where $B \subseteq C$ and you'll get a counter-example") is actually worse for your reader (and strictly speaking not a complete proof) even though it seems to give your reader more information.

You must give them a particular example; leaving it as "here's how to make an example" is incorrect – imagine you said "any prime number that is greater than 100 and even is a counter-example." There are no such numbers! Someone reading your proof has to verify not just your (now more abstract) proof of all your counter-examples, they also have to check the object you describe actually exists. Showing an object with the properties you claim exists is your job as someone proving an existential.

If your proof of an $\exists$ (or equivalently, disproof of a $\forall$) does not have a particular counter-example, it's not a full proof.

So how to avoid these errors?

(a) Make sure you understand exactly what the claim is asking you to show. If you aren't showing something is true $\forall a$, think carefully about whether you actually want/need that variable to be arbitrary.

(b) When you're done with a proof, read it through – have you told us whether each variable is arbitrary or just one that exists?

(c) When you read through your final proof, make sure that variables you introduced as arbitrary really are.