CSE 311 : Final Exam
Released Friday, March 12, 12:01 AM
Due Thursday, March 18, 11:59 PM

Notes

- Do not collaborate with anyone besides your prescribed Canvas group.
- The same rules apply as to your homework: you may discuss the problems together, but you must write up your solutions separately, without notes or other artefacts of your discussions.
- You may refer to course materials but no other external materials (no other websites, textbooks, etc). If you unintentionally run into materials relevant to this exam, please cite them.
- The extra credit problem will be worth more substantial points than the usual homework extra credit, worth between $.5$ and 1 of the other problems.

1. Induction: a recurrent topic

Consider the following recursively defined function:

$$T(n) = \begin{cases} 3 & \text{if } n = 1 \\ 6 & \text{if } n = 2 \\ 18 & \text{if } n = 3 \\ T(n-3)(n^3 - 3n^2 + 2n) & \text{Otherwise} \end{cases}$$

Use induction to prove that that $T(n) = 3(n!)$ for all $n \geq 1$.

2. Putting it all together

Consider the alphabet $\Sigma = \{0, 1, \diamond\}$.

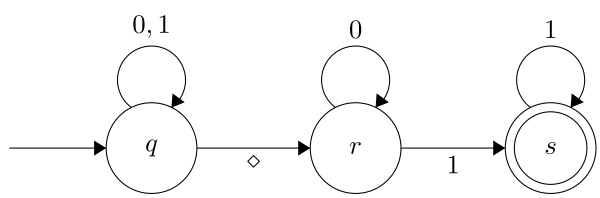
- Let A be the set of strings x from Σ^* such that the following formula holds, where CharAt(x, y, z) means that $x \in \Sigma^*$, $y \in \mathbb{N}$, $z \in \Sigma$ and z is the y-th character in x.

  $$\forall i\, \forall j\, \forall k\, \forall c\, \big(\text{CharAt}(x, i, \diamond) \land \text{CharAt}(x, k, c) \land i < k\big) \rightarrow \big(c \neq \diamond \land [(i < j \land j < k \land \text{CharAt}(x, j, 1)) \rightarrow c \neq 0]\big)$$

- Let B be the set of strings generated by the following CFG.

  $$\begin{aligned} S &\rightarrow A \mid A \diamond M \\ A &\rightarrow 0A \mid 1A \mid \varepsilon \\ M &\rightarrow 0M \mid M1 \mid MM \mid \varepsilon \end{aligned}$$

- Let C be the set of strings accepted by the following NFA.



(a) Determine, with proof, whether the sets A and B are equal.

(b) Determine, with proof, whether the sets A and C are equal.

3. Structurally Sound

Prove the regular expression `(3*1)*(1*3)*` matches the same language as `(3 ∪ 1 ∪ 1)*`, using structural induction on strings over $\Sigma = \{[0-9], [A-Z], [a-z]\}$.

4. Extra credit: Em-POWER-ed Sets

Let $\mathcal{P}(A)$ represent the powerset of set A. We can easily argue that $\mathcal{P}(\mathbb{N})$ is bijective with $\mathbb{R}$. The proof for this is the standard diagonalization proof for uncountable sets.

Now, suppose I had some set A such that $\mathcal{P}(A)$ is bijective with $\mathbb{N}$. Describe this set A or prove that no such set can exist.