Section 01: Propositional Logic Translation

1. Warm-Up

Translate the English sentences below into symbolic logic.

(a) If I am lifting weights this afternoon, then I do a warm-up exercise.

(b) If I am cold and going to bed or I am two-years old, then I carry a blanket.

2. Truth Tables

Write a truth table for each of the following:

(a) \((r \oplus q) \lor (r \oplus \neg q)\)

(b) \((r \lor q) \rightarrow (r \oplus q)\)

(c) \(p \leftrightarrow \neg p\)

3. If I can translate, then...

For each of the following more obscure English ways to write an implication, define atomic propositions and write a symbolic representation of the sentence.

(a) whenever I walk my dog, I make new friends.

(b) I will drink coffee, if Starbucks is open or my coffeemaker works.

(c) Being a U.S. citizen and over 18 is sufficient to be eligible to vote.

(d) I can go home only if I have finished my homework.

(e) Having an internet connection is necessary to log onto zoom.

4. I can rewrite these formulas in English, only if...

Given propositions and a logical formula, write two potential English translations. The meanings of the sentences will be the same (They represent the same formula!), but they can still look quite different.

(a) \(p\): The sun is out
   \(r\): We have class outside

   \(p \rightarrow r\)

(b) \(a\): the book has been out for a week.
   \(b\): I don't have homework.
   \(c\): I have finished reading the book.

   \((a \land b) \rightarrow c\)
(c) \( p \): I have read the manual  
\( r \): I operate the machine  
\( r \rightarrow p \)

5. Translation

For each of the following, define propositional variables and translate the sentences into logical notation.
(a) I will remember to send you the address only if you send me an e-mail message.
(b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.
(c) Unless I am trying to type something, my cat is either eating or sleeping.

6. Tea Time

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation.
(b) Fill out a truth table for your expression.

7. Exclusive Or

Exclusive or \( (\oplus) \) and inclusive or \( (\lor) \) both can be translated as “or” in English. For each of the following ambiguous phrases, decide which type of “or” is likely meant and why.
(a) Experience with C or Java is required.
(b) Lunch includes soup or salad.
(c) Publish or perish.
(d) To enter the country, you need a passport or voter registration card.

8. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.
(a) \( p \rightarrow r \) vs. \( r \rightarrow p \)
(b) \( a \rightarrow (b \land c) \) vs. \( (a \rightarrow b) \land c \)
9. They mean the same thing

In the activity from lecture 2, we showed the following.

\[ \neg(q \to r) \equiv \neg(\neg q \lor r) \]

Use the elementary equivalences presented at the end of lecture 2 to argue that the following pairs are equivalent.

\[ \neg(\neg q \lor r) \equiv \neg(\neg q) \land \neg r \quad (1) \]

\[ \neg(\neg q) \land \neg r \equiv q \land \neg r \quad (2) \]

\[ q \land \neg r \equiv \neg r \land q \quad (3) \]

Your friend says this means that \( \neg(q \to r) \equiv \neg r \land q \). Is that true?

10. Equivalent Translations

Prove that the following English statements are equivalent.

(i) Unless it isn't raining or I don't have an umbrella, I buy a book.
(ii) It isn't raining or I don't have an umbrella or I buy a book.

11. Circuitous

Translate the following circuit into a logical expression.