CSE 311: English Proof Style Guide

1. Be careful in your introduction

- Remember to introduce your variables.
- If you are trying to prove a $\forall$, you will need to introduce the variable as arbitrary.
- Even if the original claim doesn't explicitly say for all, if it's implicitly a universal claim, your proof should explicitly call it arbitrary.
- If the variable's type will be relevant state its type (e.g. if you're going to use x in a divides statement, it needs to be an integer).
- If you're proving an implication, be sure to suppose the hypothesis of the implication as soon as you've introduced your variables.
- If you're doing proof by induction, remember to define the $P()$ you are proving.

2. Identify your proof technique

If you're doing anything other than a direct proof (i.e. proving $p \to q$ by supposing p and deriving q or showing p by starting from a known statement and deriving p) then you must state your proof technique.

- If you're doing proof by contrapositive, start by saying "We argue by contrapositive" (or equivalent).
- If you're doing proof by contradiction, start by "suppos[ing] for the sake of contradiction" (or equivalent).
- If you're doing proof by cases, say "We split into cases" (or similar) and clearly label the cases.
- If you're doing proof by induction, say "We argue by induction on n" (or "by structural induction") as appropriate.

3. Don't forget a conclusion

Most of our proof techniques have some sort of concluding sentence.

- Proof by contradiction ends with "But that's a contradiction!" (or equivalent).
- Proof by induction ends with "by the principle of induction..."
- Just after ending cases, you'll say "in [either/every] case, we have..."

Even if your proof doesn't require a conclusion, it's rarely a bad idea to remind your reader what you've just shown.

4. In the heart of your proof

Justify your steps. Some steps (like simple algebra) you may not need to explicitly justify, but when in doubt extra justification is better than not enough.

When you're citing a theorem we've shown you, we usually have a name in the lecture or on a reference sheet. You can cite them by name, or (if we don't give you a name) by saying something like "by the claim proven in lecture X slide Y."

Be very careful about logical ordering. Do not assert a statement before you can show it. Showing your goal simplifies to a true statement isn't a rigorous argument; this is equivalent to showing if your goal, then a true statement. But that doesn't mean your goal is true (what if that implication was vacuous?). You need to show the true statement implies your goal (then your goal must be true).

5. That Word Doesn't Mean What You Think It Means

Mathematical English has some peculiarities about common words. Be sure you're using them in the way you intend.

5.1. By definition

Only say "by definition" if you are directly applying the definition of an object. Do not use "by definition" to mean "if you think about the definition for a bit, you'll see this claim."

Good example: $x^2 = 4z^2 = 2(2z^2)$. $2z^2$ is an integer, so x^2 is even.

Bad example: $A \subseteq S \cap T$ so by definition of subset and intersection $A \subseteq S \wedge A \subseteq T$.

The definition of subset would let you say

$$\forall x(x \in A \to x \in S \cap T)$$

And the definition of intersection would let you say:

$$\forall x(x \in A \to [x \in S \wedge s \in T])$$

That isn't the statement $A \subseteq S \wedge A \subseteq T$. It's certainly closer than the starting statement! But it's too much to just call this "by definition."

5.2. Arbitrary

Don't insert the word arbitrary arbitrarily. You can introduce a variable as arbitrary when you create it. Once you do any modification to that variable, the result is no longer arbitrary. Combining two arbitrary variables does not give an arbitrary variable.

If a variable truly is arbitrary then you should be able to correctly plu in any value in your domain and have the rest of the proof work.

Good example: Let (a, b) be an arbitrary element of $A \times B$, by definition of cross product $a \in A$ and $b \in B$.

Bad example: Let a be an arbitrary element of A and b be an arbitrary element of B. Then (a, b) is an arbitrary element of $A \times B$.

Good example: Let x be an arbitrary integer. $x + 1$ is a positive integer.

Bad example: Let x be an arbitrary natural number and let y be an arbitrary natural number. Since x and y were arbitrary, $x + y$ is arbitrary natural number.

5.3. Random

Do not use "random" as a synonym for "arbitrary." They are not synonyms. Random means "I rolled a die or flipped a coin and got this value." Arbitrary means "anything you like can be plugged in here and it'll work." If you want to make a statement for all x, a random x does not suffice, you need a random one.