CSE 311: Foundations of Computing I

Lecture 1: Propositional Logic
Today’s Agenda

• Course Goals
• Administrivia
• First topic (Propositional Logic)
About CSE 311
Course Goals

1. Teach you the theory background needed for other CSE courses
   – only topics used in many areas of CSE

2. Teach you how to make and communicate rigorous and formal arguments
   – want to know for certain that systems work

3. Introduce you to theoretical CS
   – may be the only theory course you take
Course Content

We will study the *theory* needed for CSE:

**Logic:**
- How can we describe ideas *precisely*?

**Proofs:**
- How can we be *positive* we’re correct?

**Number Theory:**
- How do we keep data *secure*?

**Sets & Relations:**
- How do we store and describe information?

**Finite State Machines:**
- How do we design hardware and software?

**General Computing Machines:**
- Are there problems computers *can’t* solve?
Some Perspective

Computer Science and Engineering

Programming

CSE 14x

Theory

CSE 311

Hardware
About the Course

Become a better programmer

By the end of the course, you will have the tools to...

• reason about difficult problems
• automate difficult problems
• communicate ideas, methods, objectives
• understand fundamental structures of CS
About the Course

Become more comfortable with formal methods

Difficult problems often require formalism (“math”)
  • don’t confuse correlation with causation

Formalism is a tool we apply when problems get difficult
Okay, now listen up. Nobody gets in here without answering the following question: A train leaves Philadelphia at 1:00 p.m. It's traveling at 65 miles per hour. Another train leaves Denver at 4:00... Say, you need some paper?
Using Algebra to Solve Problems

• It’s 1720 miles from Philadelphia to Denver. A train leaves Philly going 65 mph. Three hours later, a train leaves Denver at 40 mph. At what time do they collide?

• Let t be the time travelled by the Philly train
  – Philly train has traveled 65t miles
  – Denver train has travelled 40(t + 3) miles
  – Collide when 65t + 40(t + 3) = 1720
  – Solve for t
About the Course

And become more comfortable with formal methods

Formalism is a tool we apply when problems get difficult

• helps us get through without making mistakes
• turns confusing English into precise math
• sometimes even gives “turn the crank” solutions
• algebra to find t is mechanical

formalism is our instrument panel
(needed for the difficult conditions)
Administrivia
Instructors

Kevin Zatloukal

Section A
MWF 10:30-11:20 in GUG 220

Section B lectures will be recorded

Kevin Zatloukal

Section B
MWF 1:30-2:20 in CSE2 G20

Office Hours:
M 12:00-1:00 in CSE 436
F 4:30-5:20 via Zoom
TAs

Varun Agrawal  Mengyi Shan
Mrigank Arora  David Shiroma
Linden Gan  Helena Stafford
Shreya Jayaraman  Siddharth Vaidyanathan
Ben Lambert  Jason Waataja
Audrey Ma  Ivy Wang
Melissa Mitchell  Alice Wang
Long Nguyen  Zedong Wu
Andrey Risukhin  Ben Zhang

Quiz Sections

• every Thursday
• led by 1-2 TAs

Office Hours

• multiple hours a day all week
• see web site for times and locations
CSE 311: Foundations of Computing I

The goals of this course are (1) to teach you the mathematical background needed for upper-level CSE courses and (2) to introduce you to theoretical computer science, including the critical concept of a mathematical proof.

Staff

Instructor: Kevin Zatloukal (kevinz at cs)

TAs: Varun Agrawal, Mrigank Arora Linden Gan, Philip Garrison, Siddharth Vaidyanathan, Shreya Jayaraman, Ben Lambert, Audrey Ma, Melissa Mitchell, Long Nguyen, Andrey Risukhin, Mengyi Shan, David Shiroma, Helena Stafford, Jason Waataja, Ivy Wang, Alice Wang, Zedong Wu, and Ben Zhang

Contact: Please use the message board whenever possible. The answer to your question is likely to be helpful to others in the class, and, by using the message board, the answer be available to them as well. For other private matters, send email to cse311-staff at cs, which will reach both the instructor and TAs.

Activities

Course activities will be a mix of in-person and online.

Lectures: We will have in-person lectures on Monday, Wednesday, and Friday. These will take place at 10:30-11:20am in GUG 220 for Section A and 1:30-2:20pm in CSE2 G20 for Section B.

Make sure you read the syllabus fully
Communication

Course mailing list (auto-subscribed)
- for important course announcements from me
- e.g., changes to homework problems or due dates
- used infrequently but do check your UW email

Ed message board (link on web site)
- best way to ask questions

Staff mailing list (cse311-staff at cs)
- for private matters
- goes to myself and the TAs
Exams

• Midterm exam will be in-class, late in the quarter
  – will spill almost all the details ahead of time
  – how many problems, their format, etc.

• Final exam during finals week
  – Section B at the listed time place (Mon 2:30, CSE2 G20)
  – Section A at an unusual time & place:
    same room as Section B
    held immediately after Section B’s final
    (Mon 4:30, CSE2 G20)
About grades...

• Grades were very important up until now

• Grades are much less important going forward
  – companies care much more about your interviews
  – grad schools care much more about recommendations

• Understanding the material is much more important
  – interviews test your knowledge from 300-level classes
  – good recommendations involve knowledge beyond the classes

• Please relax and focus on learning as much as possible
  – all the 300-level material will be useful in your career
Please focus less on points

• Most time spent on questions about grading issues is not worthwhile to either the student or teacher

• Try to avoid asking “will I lose points if…”

• If the thought of losing points worries you, show more work
  – no sense having a 30-minute discussion to save 10 minutes

• Ideally, I only look at individual grades 1 day per quarter
CSE courses can be hard

- Not my intention
  - I’ll try to make this as stress free as possible
  - But...

- You have a lot to learn
  - can’t yet solve the problems you’ll need to
  - we will move quickly

- You need a lot more practice
Collaboration Policy

• Collaboration with others is encouraged!

• Basic idea:
  – do help other students learn
  – do not help other students avoid learning

• Policy: you must write up your own solution
  – your solutions are not group work
  – you must list your collaborators
Collaboration Policy

• Collaboration with others is encouraged!

• Important rules when working together:
  – do not leave with any solution written down or photographed
  – wait 30 minutes before writing up your solution

• You cannot “collaborate” with Google, MathOverflow, etc.

• See Allen School Academic Misconduct policy
  – serious consequences for cheating (e.g., expulsion)

• No scenario where it is necessary to share your write up
  – if you can’t fully solve a problem in time, you’ll just lose some points
Study Groups

• Will send out a form to fill out if you want to be matched with other students into a study group
  – form should be coming tomorrow
  – match based on available time & frequency
  – about 4 people per group
Propositional Logic
What is logic and why do we need it?

Logic is a language, like English or Java, with its own
• words and rules for combining words into sentences (syntax)
• ways to assign meaning to words and sentences (semantics)

Why learn another language?
We know English and Java already?
Why not use English?

– Turn right here...
  Does “right” mean the direction or now?

– We saw her duck
  Does “duck” mean the animal or crouch down?

– Buffalo buffalo Buffalo buffalo buffalo
  buffalo Buffalo buffalo
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

Natural languages can be unclear or imprecise
Why learn a new language?

We need a language of reasoning to
  – state sentences more precisely
  – state sentences more concisely
  – understand sentences more quickly

Formal logic has these properties
Propositions: building blocks of logic

A *proposition* is a statement that

– is either true or false

– is “well-formed”
A *proposition* is a statement that
- is either true or false
- is “well-formed”

All cats are mammals
  true

All mammals are cats
  false
Are These Propositions?

2 + 2 = 5
   This is a proposition. It’s okay for propositions to be false.

x + 2 = 5389, where x is my PIN number
   This is a proposition. We don’t need to know what x is.

Akjsdf!
   Not a proposition because it’s gibberish.

Who are you?
   This is a question which means it doesn’t have a truth value.

Every positive even integer can be written as the sum of two primes.
   This is a proposition. We don’t know if it’s true or false, but we know it’s one of them!
Propositions

We need a way of talking about *arbitrary* ideas...

Propositional Variables: $p, q, r, s, ...$

Truth Values:
- $T$ for *true*
- $F$ for *false*
Familiar from Java

• Java `boolean` represents a truth value
  – constants `true` and `false`
  – variables hold *unknown* values

• Operators that calculate new truth values from given ones
  – unary: `not` (!)
  – binary: `and` (&&), `or` (||)
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<thead>
<tr>
<th>Logical Connectives</th>
<th>Symbol(s)</th>
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<tbody>
<tr>
<td>Negation (not)</td>
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## Some Truth Tables

### Truth Table for $p \land \neg p$

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Logic forces us to distinguish \( \lor \) from \( \oplus \)
Implication

“If it’s raining, then I have my umbrella”

It’s useful to think of implications as promises. That is “Did I lie?”

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The only lie is when:

(a) It’s raining AND
(b) I don’t have my umbrella
Implication

“If it’s raining, then I have my umbrella”

Are these true?

2 + 2 = 4 → earth is a planet

The fact that these are unrelated doesn’t make the statement false! “2 + 2 = 4” is true; “earth is a planet” is true. T→T is true. So, the statement is true.

2 + 2 = 5 → 26 is prime

Again, these statements may or may not be related. “2 + 2 = 5” is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to *understand* what this proposition means.
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to understand what this proposition means.

First find the simplest (atomic) propositions:

- \( q \) “Garfield has black stripes”
- \( r \) “Garfield is an orange cat”
- \( s \) “Garfield likes lasagna”

\( (q \text{ if } (r \text{ and } s)) \text{ and } (r \text{ or } (\text{not } s)) \)
### Logical Connectives

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$q$ “Garfield has black stripes”  
$r$ “Garfield is an orange cat”  
$s$ “Garfield likes lasagna”  

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”  

$(q$ if $(r$ and $s))$ and $(r$ or $(\neg s))$
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\[
(q \text{ if } (r \text{ and } s)) \text{ and } (r \text{ or } \neg s)
\]

\[
((r \land s) \rightarrow q) \land (r \lor \neg s)
\]