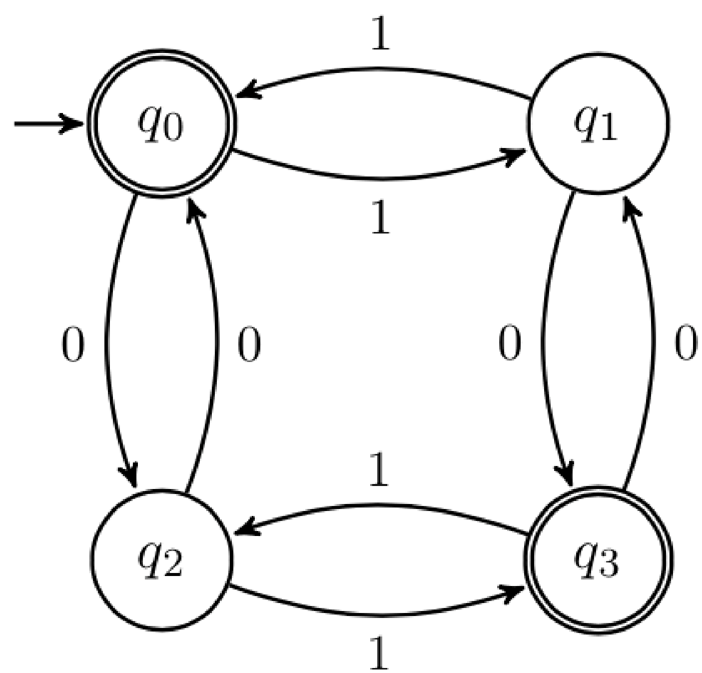
CSE 311: Foundations of Computing

Lecture 20: Directed Graphs, Closures, & Finite State Machines



Last Class: Relations & Composition

Let A and B be sets,
A **binary relation from** A **to** B is a subset of $A \times B$

Let A be a set,
A **binary relation on** A is a subset of $A \times A$

The composition of relation R and S, $S \circ R$ is the relation defined by:

$$S \circ R = \{ (a, c) \mid \exists\, b \text{ such that } (a,b) \in R \text{ and } (b,c) \in S \}$$

Last Class: Powers of a Relation

$\boldsymbol{R^0} \quad = \{(\boldsymbol{a}, \boldsymbol{a}) \mid \boldsymbol{a} \in \boldsymbol{A}\}$ “the equality relation on $\boldsymbol{A}$”

$\boldsymbol{R^{n+1}} = \boldsymbol{R^n} \circ \boldsymbol{R}$ for $\boldsymbol{n} \geq \boldsymbol{0}$

Last class: Matrix Representation

Relation $\boldsymbol{R}$ on $\boldsymbol{A} = \{a_1, \dots, a_n\}$

$$\boldsymbol{m_{ij}} = \begin{cases} 1 & \text{if } (a_i, a_j) \in \boldsymbol{R} \\ 0 & \text{if } (a_i, a_j) \notin \boldsymbol{R} \end{cases}$$

{ (1, 1), (1, 2), (1, 4), (2, 1), (2, 3), (3, 2), (3, 3), (4, 2), (4, 3) }

	1	2	3	4
1	1	1	0	1
2	1	0	1	0
3	0	1	1	0
4	0	1	1	0

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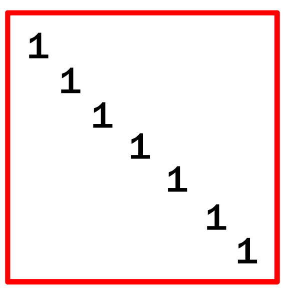
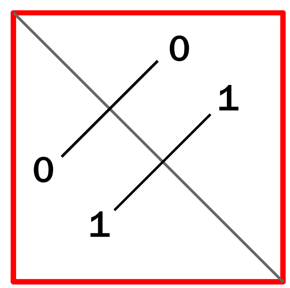


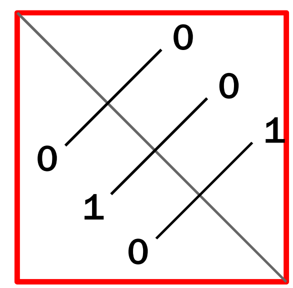
reflexive

$\leq, =$



symmetric

$=, \equiv$



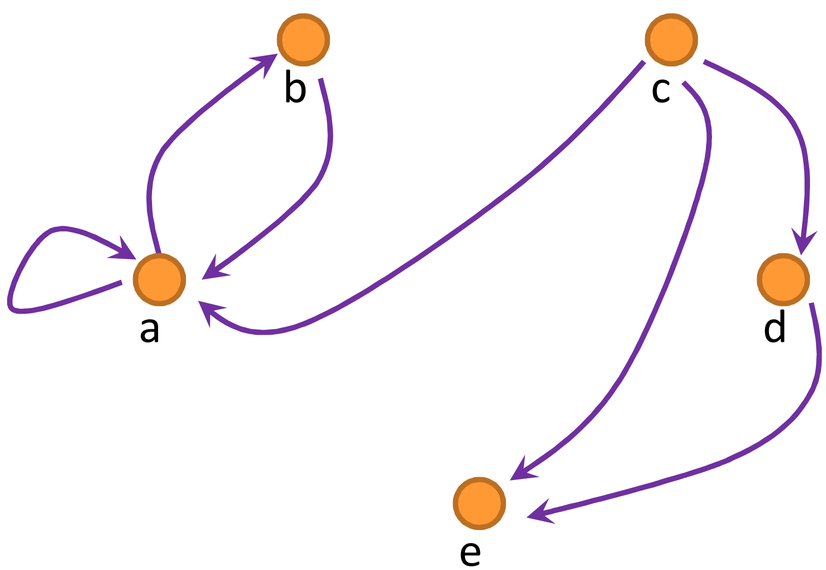
antisymmetric

$\leq, <$

Last Class: Representation of Relations

Directed Graph Representation (Digraph)

{(a, b), (a, a), (b, a), (c, a), (c, d), (c, e) (d, e) }



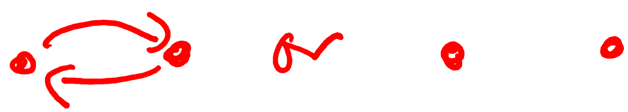
How Properties of Relations show up in Graphs

Let R be a relation on A.

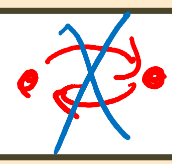
R is **reflexive** iff $(a,a) \in R$ for every $a \in A$

"self-loop" on every node

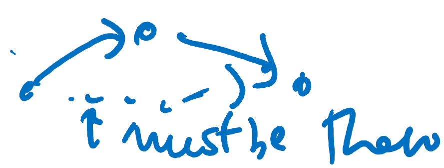
R is **symmetric** iff $(a,b) \in R$ implies $(b, a) \in R$



R is **antisymmetric** iff $(a,b) \in R$ and $a \neq b$ implies $(b,a) \notin R$



R is **transitive** iff $(a,b) \in R$ and $(b, c) \in R$ implies $(a, c) \in R$



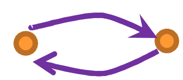
How Properties of Relations show up in Graphs

Let R be a relation on A.

R is **reflexive** iff $(a,a) \in R$ for every $a \in A$


at every node

R is **symmetric** iff $(a,b) \in R$ implies $(b, a) \in R$


or


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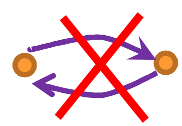

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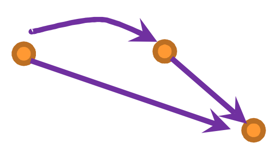
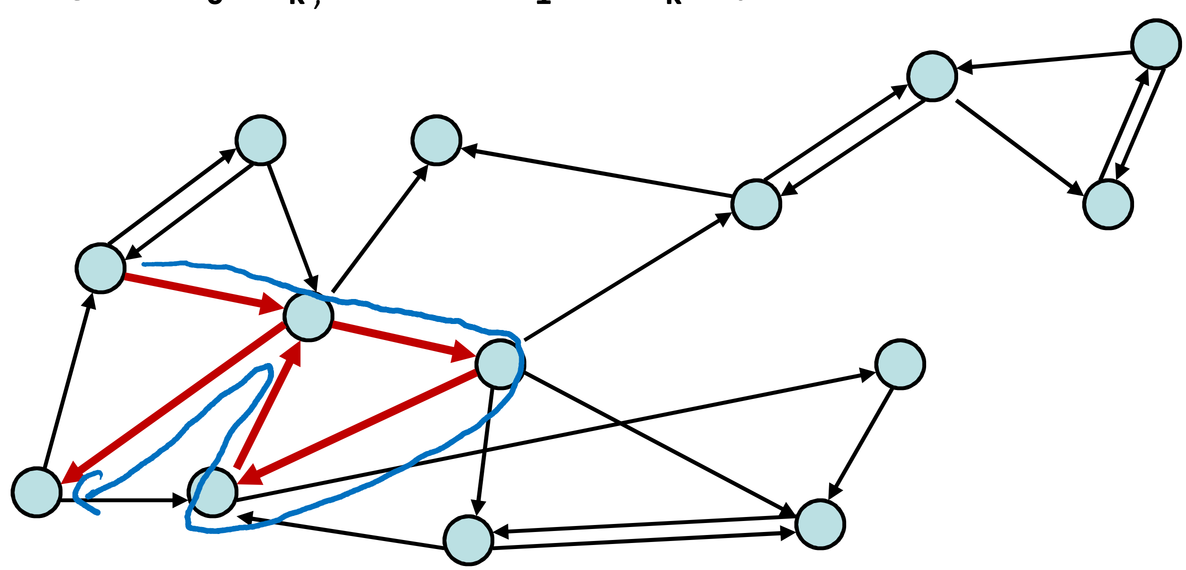


Directed Graphs

G = (V, E)   V – vertices
E – edges, ordered pairs of vertices $E \subseteq V \times V$

Path: $v_0, v_1, \ldots, v_k$ with each (v_i, v_{i+1}) in E

Simple Path: none of $\mathbf{v_0}, \ldots, \mathbf{v_k}$ repeated
Cycle: $\mathbf{v_0} = \mathbf{v_k}$
Simple Cycle: $\mathbf{v_0} = \mathbf{v_k}$, none of $\mathbf{v_1}, \ldots, \mathbf{v_k}$ repeated



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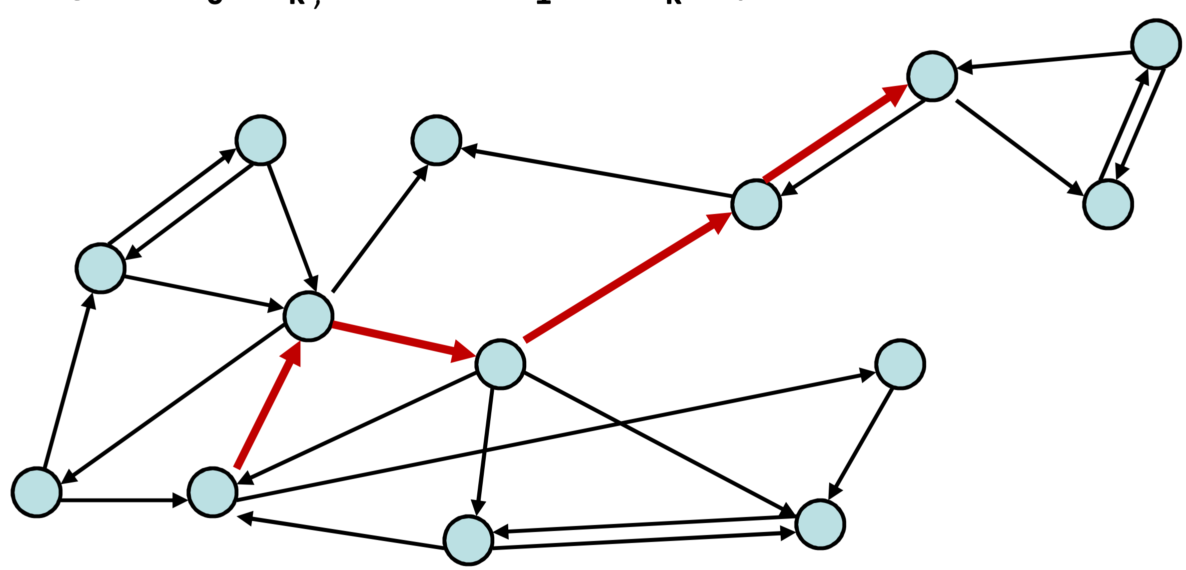

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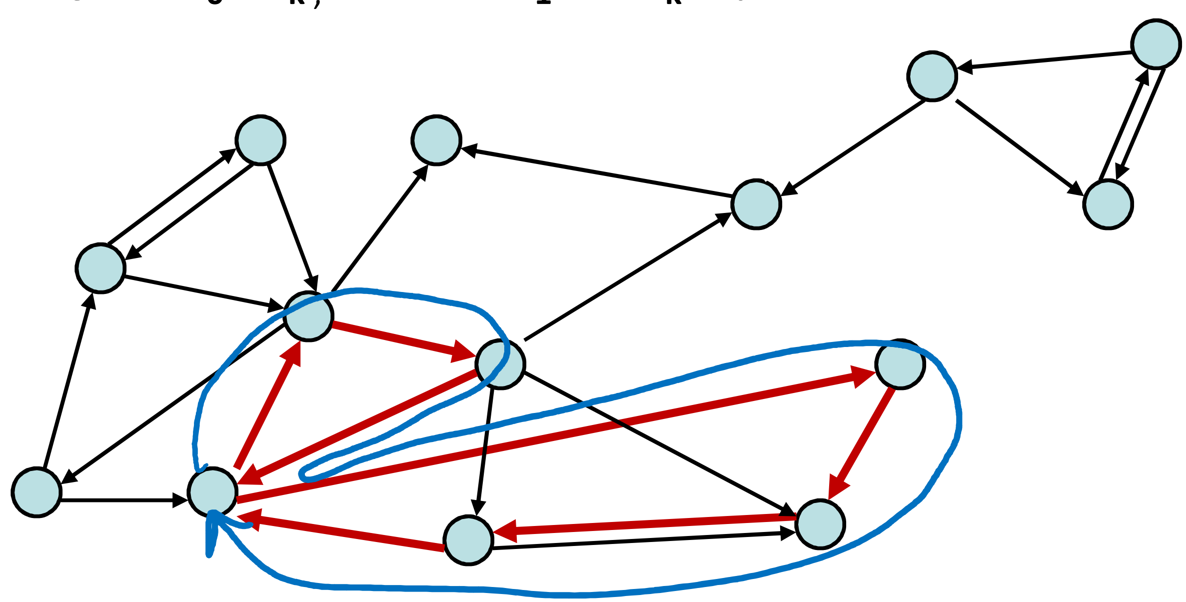

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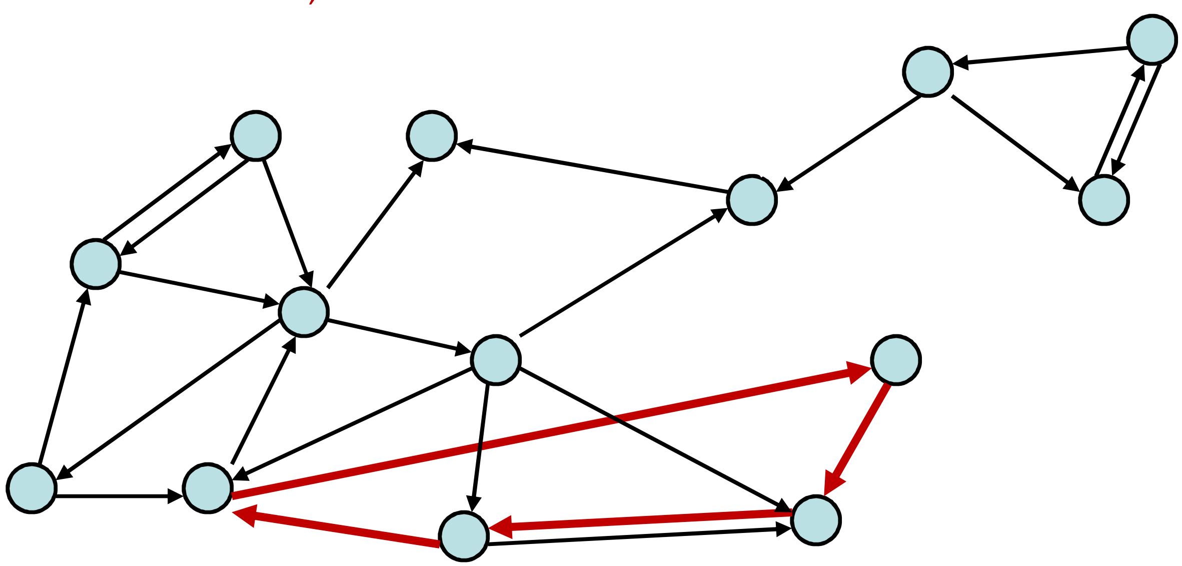
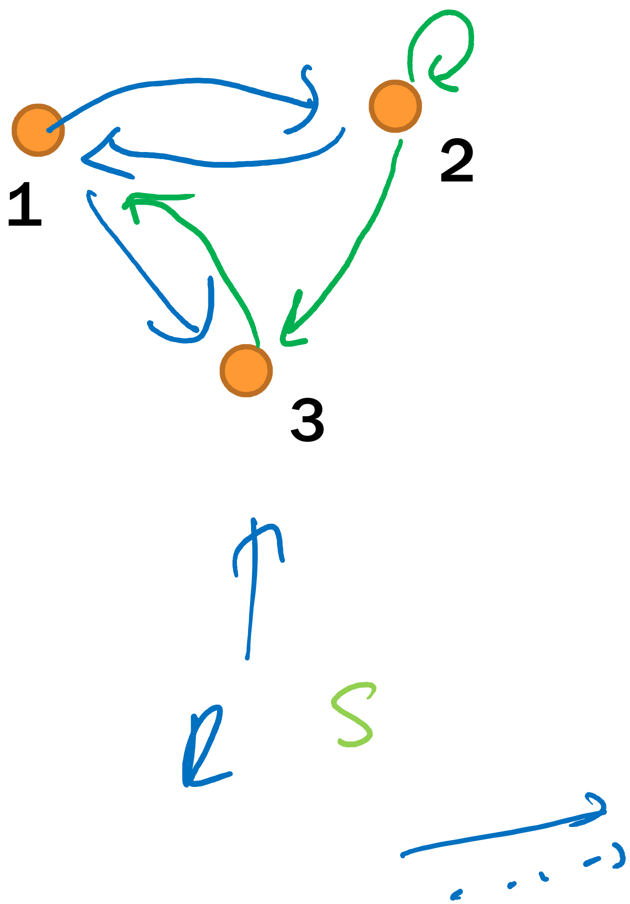
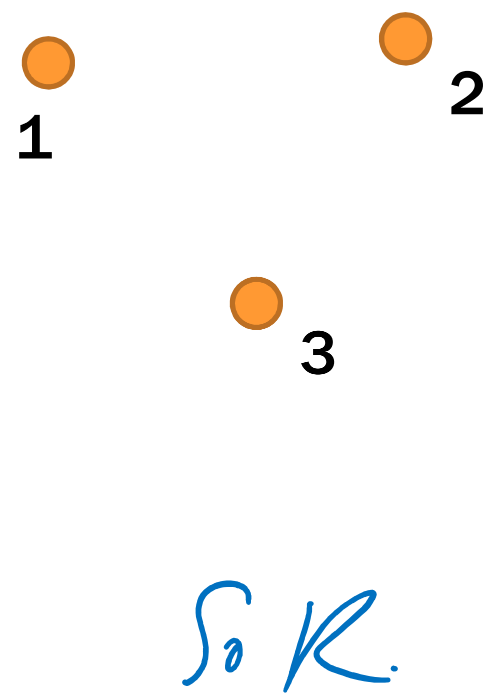


Relational Composition using Digraphs

If $S = \{(2,2),(2,3),(3,1)\}$ and $R = \{(1,2),(2,1),(1,3)\}$

Compute $S \circ R$





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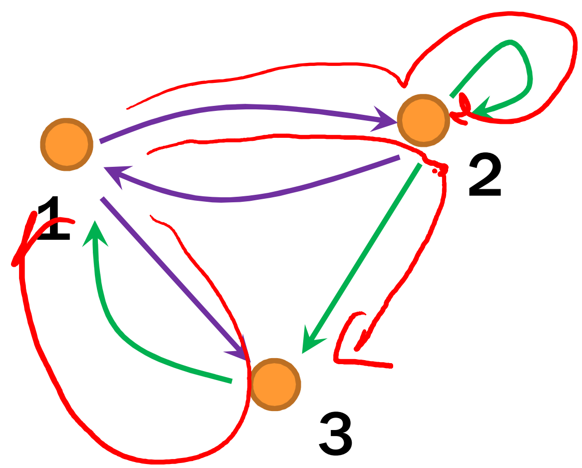

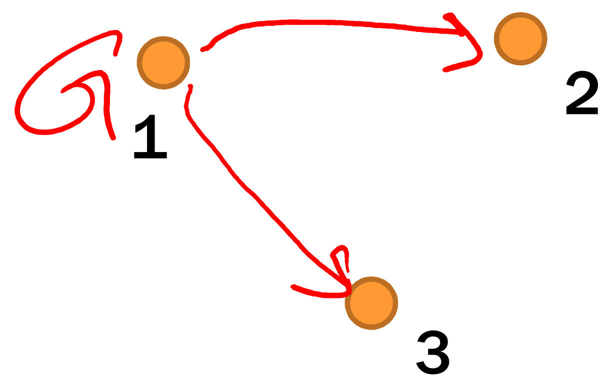

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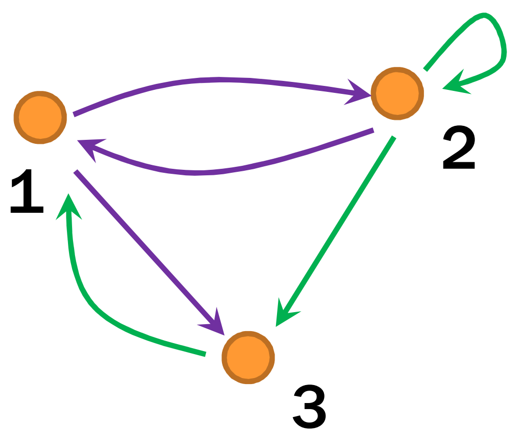

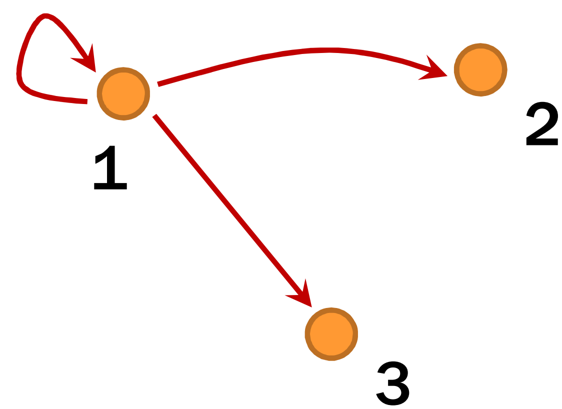

Paths in Relations and Graphs

Defn: The **length**

Let $\boldsymbol{R}$ be a relation on a set $\boldsymbol{A}$. There is a path of length $\boldsymbol{n}$ from **a** to **b** if and only if (**a,b**) $\in \boldsymbol{R}^{\boldsymbol{n}}$

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