CSE 311: Foundations of Computing I

Lecture 1: Propositional Logic
About CSE 311
Some Perspective

Computer Science and Engineering

Programming

CSE 14x

Theory

CSE 311

Hardware
About the Course

We will study the theory needed for CSE:

Logic:
  How can we describe ideas precisely?

Formal Proofs:
  How can we be positive we’re correct?

Number Theory:
  How do we keep data secure?

Relations/Relational Algebra:
  How do we store information?

Finite State Machines:
  How do we design hardware and software?

Turing Machines:
  Are there problems computers can’t solve?
About the Course

Will help you become a better programmer

By the end of the course, you will have the tools for:

• reasoning about difficult problems
• automating difficult problems
• communicating ideas, methods, objectives

and will understand fundamental structures of CS
Course Logistics
Instructor

Paul Beame

MWF 1:30-2:20 in CSE2 G01

Office Hours (tentative):
M 2:30-4:00 and WF 2:30-3:00 in CSE 668
TAs

Teaching Assistants:

Siddharth Iyer  Josh Shin
Suraj Jagadeesh  Xiaoyue Sun
Karishma Mandyam  Jason Waataja

Section:
Thursdays  – starting this week

Office Hours: TBD

(Optional) Book:
Rosen: Readings for 6th or 7th editions.
Many used copies available
Good for practice with solved problems
# CSE 311: Foundations of Computing I

**Winter, 2020**

**Paul Beame**  
MWF 1:30-2:20, CSE 201  
Office hours: TBA  
CSE 669  

**Email and discussion:**  
e-mail list: cse311a_email [archived]  
Please send any e-mail about the course to cse311-staff@cs.washington.edu  

**Textbook:**  
There is no required text for the course. Especially over the first 6-7 weeks of the course, the following textbook can be a useful companion: *Rosen, Discrete Mathematics and Its Applications*, Mcgraw-Hill. There are many editions of this book, and lots of used copies available; new copies are extremely expensive. A copy should be available on short-term loan from the Engineering Library.

## Lectures

<table>
<thead>
<tr>
<th>#</th>
<th>date</th>
<th>topic</th>
<th>slides</th>
<th>linked</th>
<th>reading (Rosen)</th>
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<tbody>
<tr>
<td>1</td>
<td>Mon, Jan 6</td>
<td>Propositional Logic</td>
<td>1.1, 1.2</td>
<td>7th</td>
<td>6th</td>
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<tr>
<td>2</td>
<td>Wed, Jan 8</td>
<td>Logical Equivalence/Gates</td>
<td>1.1-1.3</td>
<td>7th</td>
<td>6th</td>
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<td>3</td>
<td>Fri, Jan 10</td>
<td>More Logic/Circuits</td>
<td>1.2-1.3</td>
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<td>11.1-1.13</td>
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<td>1.2-1.3</td>
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<td>11.1-1.13</td>
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<td>Wed, Jan 15</td>
<td>Canonical Forms, Predicate Logic</td>
<td>1.4-1.5</td>
<td>7th</td>
<td>1.3-1.4 6th</td>
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<td>Fri, Jan 17</td>
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<td>Mon, Jan 20</td>
<td>Martin Luther King Day NO CLASS</td>
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<td>Wed, Jan 22</td>
<td>Logical Inference and Proofs</td>
<td>1.6-17</td>
<td>7th</td>
<td>1.5-1.7 6th</td>
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<td>Predicate Logic Proofs</td>
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<td>10</td>
<td>Mon, Jan 27</td>
<td>Set Theory</td>
<td>2.1-2.3</td>
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<td>11</td>
<td>Wed, Jan 29</td>
<td>Modular Arithmetic</td>
<td>4.1-2.4</td>
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<td>3.4-2.5 6th</td>
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<td>12</td>
<td>Fri, Feb 3</td>
<td>Applications of Mod, Number Theory/Factoring</td>
<td>4.1-3.7</td>
<td>7th</td>
<td>3.4-2.6 6th</td>
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<tr>
<td>13</td>
<td>Mon, Feb 6</td>
<td>GCD Euclid’s Algorithm, Modular Equations</td>
<td>4.3-4.7</td>
<td>7th</td>
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Work

Homework:
Due WED at 11:00 pm online (Gradescope)
Write up individually
Extra Credit

Exams:
Midterm in class on Friday, Feb 14
Final exam:
Monday, March 16  2:30-4:20 pm

Grading (roughly):
50% Homework
15-20% Midterm
30-35% Final Exam
Communication

• You are already on the class e-mail list
  – Major announcements here, archive reachable from the course webpage

• If you want to email to us (me & TAs):
  cse311-staff@cs.washington.edu

• Discussion board
  – accept invitation to Ed class discussion board
About grades...

- Grades were very important up until now...
About grades...

- Grades were very important up until now

- Grades are much less important going forward
  - companies care much more about your interviews
  - grad schools care much more about recommendations
About grades...

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• Understanding the material is much more important
  – interviews test your knowledge from these classes
  – good recommendations involve knowledge beyond the classes
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• Understanding the material is much more important
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• Please relax and focus on learning
Please calm down about grades

• Most time spent on questions about grading issues is not worthwhile to either the student or teacher

• Try to avoid asking “will I lose points if...”

• If the thought of losing points worries you, show more work
  – no sense having a 30 minute discussion to save 10 minutes
Collaboration Policy

• Collaboration with others is encouraged
• BUT you must:
  – list anyone you work with
  – turn in only your own work

• Recommended approach for group work
  – do not leave with any solution written down or photographed
  – wait 30 minutes before writing up your solution

• See Allen School Academic Misconduct policy also
No Late Days

• To be accepted, late submission (with good reason) must be arranged in advance 48 hours before the deadline
If you are worried about Mathy aspects of 311

- Associated 1-credit CR/NC workshop
  - CSE 390ZA (not yet available for enrollment)
  - Extra collaborative practice on 311 concepts, study skills, a small amount of assigned work
  - 1.5 hours Thursdays 3:30 pm
  - Full attendance is required, else NC
  - NOT for help with 311 homework

- Anyone in 311 can sign up but enrollment is limited
Getting used to being formal

As problems we deal with get harder we need stronger tools...

Formalism is a tool we apply when problems get difficult

– helps us get through without making mistakes
– sometimes even gives “turn the crank” solutions
Propositional Logic
What is logic and why do we need it?

Logic is a language, like English or Java, with its own
- words and rules for combining words into sentences (syntax)
- ways to assign meaning to words and sentences (semantics)

Why learn another language when we know English and Java already?
Why not use English?

– Turn right here...

– Buffalo buffalo Buffalo buffalo buffalo buffalo buffalo buffalo Buffalo buffalo

– We saw her duck
Why not use English?

- Turn right here...
  Does “right” mean the direction or now?

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo buffalo Buffalo
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.”

- We saw her duck
  Does “duck” mean the animal or crouch down?
Why not use English?

– Turn right here...
  Does “right” mean the direction or now?

– Buffalo buffalo Buffalo buffalo buffalo
  buffalo Buffalo buffalo
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.”

– We saw her duck
  Does “duck” mean the animal or crouch down?

Natural languages can be imprecise
Why not use Java?

What does this code do:

```java
public static boolean mystery(int x) {
    for (int r = 2; r < x; r++) {
        for (int q = 2; q < x; q++) {
            if (r*q == x)
                return false;
        }
    }
    return x > 1;
}
```
public static boolean mystery(int x) {
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Determines if x is a prime number
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        }
    }
    return x > 1;
}
```

Determines if x is a prime number

Programming languages can be verbose
Why learn a new language?

We need a language of reasoning to

– state sentences more precisely
– state sentences more concisely
– understand sentences more quickly
Propositions: building blocks of logic

A *proposition* is a statement that

- is either true or false
- is “well-formed”
Propositions: building blocks of logic

A proposition is a statement that
  – is either true or false
  – is “well-formed”

All cats are mammals
  true

All mammals are cats
  false
Are These Propositions?

2 + 2 = 5

x + 2 = 5

Akjsdf!

Who are you?

Every positive even integer can be written as the sum of two primes.
Are These Propositions?

2 + 2 = 5
    This is a proposition. It’s okay for propositions to be false.

x + 2 = 5
    Not a proposition. Doesn’t have a fixed truth value

Akjsdf!
    Not a proposition because it’s gibberish.

Who are you?
    This is a question which means it doesn’t have a truth value.

Every positive even integer can be written as the sum of two primes.
    This is a proposition. We don’t know if it’s true or false, but we know it’s one of them!
A first application of logic

“If I were to ask you out, would your answer to that question be the same as your answer to this one?”
Propositions

We need a way of talking about arbitrary ideas...

Propositional Variables: $p, q, r, s, ...$

Truth Values:

- $T$ for true
- $F$ for false
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to understand what this proposition means.
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to _understand_ what this proposition means.

First find the simplest (atomic) propositions:

- $p$ “Garfield has black stripes”
- $q$ “Garfield is an orange cat”
- $r$ “Garfield likes lasagna”
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to *understand* what this proposition means.

First find the simplest *(atomic) propositions:*

- \( p \) “Garfield has black stripes”
- \( q \) “Garfield is an orange cat”
- \( r \) “Garfield likes lasagna”

\((p \text{ if } (q \text{ and } r)) \text{ and } (q \text{ or } (\text{not } r))\)
<table>
<thead>
<tr>
<th>Logical Connectives</th>
<th>Example</th>
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<tbody>
<tr>
<td>Negation (not)</td>
<td>$\neg p$</td>
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<tr>
<td>Conjunction (and)</td>
<td>$p \land q$</td>
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<tr>
<td>Disjunction (or)</td>
<td>$p \lor q$</td>
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<td>Exclusive Or</td>
<td>$p \oplus q$</td>
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<td>Implication</td>
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“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

$$p \land q \land (p \lor (q \lor \neg r))$$
### Logical Connectives

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- \( r \) “Garfield likes lasagna”

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

\[
(p \text{ if } (q \land r)) \land (q \lor (\neg r))
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Some Truth Tables

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Implication

“If it’s raining, then I have my umbrella”

It’s useful to think of implications as promises. That is “Did I lie?”

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<th></th>
<th>It’s raining</th>
<th>It’s not raining</th>
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<tbody>
<tr>
<td>I have my umbrella</td>
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The only lie is when:

(a) It’s raining AND
(b) I don’t have my umbrella
Implication

“If it’s raining, then I have my umbrella”

Are these true?

\[ 2 + 2 = 4 \rightarrow \text{earth is a planet} \]

\[ 2 + 2 = 5 \rightarrow 26 \text{ is prime} \]

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\[
\begin{array}{c|c|c}
 p & q & p \rightarrow q \\
 T & T & T \\
 T & F & F \\
 F & T & T \\
 F & F & T \\
\end{array}
\]

\[
2 + 2 = 4 \rightarrow \text{earth is a planet}
\]

The fact that these are unrelated doesn’t make the statement false! “\(2 + 2 = 4\)” is true; “earth is a planet” is true. T \(\rightarrow\) T is true. So, the statement is true.

\[
2 + 2 = 5 \rightarrow 26 \text{ is prime}
\]

Again, these statements may or may not be related. “\(2 + 2 = 5\)” is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!
These sentences are implications in opposite directions:
$p \rightarrow q$

(1) “I have collected all 151 Pokémon if I am a Pokémon master”
(2) “I have collected all 151 Pokémon only if I am a Pokémon master”

These sentences are implications in opposite directions:
(1) “Pokémon masters have all 151 Pokémon”
(2) “People who have 151 Pokémon are Pokémon masters”

So, the implications are:
(1) *If* I am a Pokémon master, *then* I have collected all 151 Pokémon.
(2) *If* I have collected all 151 Pokémon, *then* I am a Pokémon master.
Implication:

- \( p \) implies \( q \)
- whenever \( p \) is true \( q \) must be true
- if \( p \) then \( q \)
- \( q \) if \( p \)
- \( q \) is necessary for \( p \)
- \( p \) is sufficient for \( q \)
- \( p \) only if \( q \)

\[
\begin{array}{ccc}
p & q & p \rightarrow q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]
Biconditional: \( p \leftrightarrow q \)

- \( p \) iff \( q \)
- \( p \) is equivalent to \( q \)
- \( p \) implies \( q \) and \( q \) implies \( p \)
- \( p \) is necessary and sufficient for \( q \)

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Biconditional: \( p \leftrightarrow q \)

- \( p \) iff \( q \)
- \( p \) is equivalent to \( q \)
- \( p \) implies \( q \) and \( q \) implies \( p \)
- \( p \) is necessary and sufficient for \( q \)

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<td>( p )</td>
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Back to Garfield...

$p$ “Garfield has black stripes”
$q$ “Garfield is an orange cat”
$r$ “Garfield likes lasagna”

"Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

\[(p \text{ if } (q \land r)) \land (q \lor (\neg r))\]

\[ (p \text{ “if” } (q \land r)) \land (q \lor \neg r) \]
Back to Garfield...

\[ p \quad \text{“Garfield has black stripes”} \]
\[ q \quad \text{“Garfield is an orange cat”} \]
\[ r \quad \text{“Garfield likes lasagna”} \]

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

\[ (p \text{ if } (q \text{ and } r)) \text{ and } (q \text{ or } \neg r) \]

\[ (p \text{ “if” } (q \land r)) \land (q \lor \neg r) \]

\[ ((q \land r) \rightarrow p) \land (q \lor \neg r) \]
## Analyzing the Garfield Sentence with a Truth Table

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<td>$q \lor \neg r$</td>
<td>$q \land r$</td>
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The truth table above evaluates the Garfield sentence with various truth values for $p$, $q$, and $r$. Each column represents a possible combination of truth values, and the final column shows whether $(q \land r) \rightarrow p \land (q \lor \neg r)$ holds true or false for each combination.
## Analyzing the Garfield Sentence with a Truth Table

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<th>$q \lor \neg r$</th>
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<th>$((q \land r) \rightarrow p) \land (q \lor \neg r)$</th>
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