Welcome!
About CSE 311
Some Perspective

Computer Science and Engineering

Programming
CSE 14x

Theory

Hardware

CSE 311
About the Course

We will study the *theory* needed for CSE:

**Logic:**
How can we describe ideas *precisely*?

**Formal Proofs:**
How can we be *positive* we’re correct?

**Number Theory:**
How do we keep data *secure*?

**Relations/Relational Algebra:**
How do we store information?

**Finite State Machines:**
How do we design hardware and software?

**Turing Machines:**
Are there problems computers *can’t* solve?
About the Course

Will help you become a better programmer

By the end of the course, you will have the tools for:

• reasoning about difficult problems
• automating difficult problems
• communicating ideas, methods, objectives
and will understand fundamental structures of CS
Course Logistics
Instructor

Paul Beame

MWF 1:30-2:20 in CSE2 G01

Office Hours (tentative):
M 2:30-4:00 and WF 2:30-3:00 in CSE 668
TAs

Teaching Assistants:

- Siddharth Iyer
- Suraj Jagadeesh
- Karishma Mandyam

- Josh Shin
- Xiaoyue Sun
- Jason Waataja

Section:

- Thursdays
  - starting this week

Office Hours: TBD

(Optional) Book:

Rosen: Readings for 6th or 7th editions.
Many used copies available
Good for practice with solved problems
Course Webpage

CSE 311: Foundations of Computing I
Winter, 2020

Paul Beame

MWF 1:30-2:20, CSE 201
Office hours: TBA
CSE 660

Email and discussion:
email list: cse311a, wiki [archived]
Please send any e-mail about the course to cse311-staff@cs.

Textbook:
There is no required text for the course. Especially over the first 6-7 weeks of
the course, the following textbook can be a useful companion: Rosen, Discrete
Mathematics and Its Applications, McGraw-Hill. There are many editions of
this book and lots of used copies available; new copies are extremely expensive.
A copy should be available on short-term loan from the Engineering Library.

Lectures

<table>
<thead>
<tr>
<th>#</th>
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<th>topic</th>
<th>slides</th>
<th>linked</th>
<th>reading (Rosen)</th>
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<td>Propositional Logic</td>
<td>1.1, 1.2</td>
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<td>2</td>
<td>Wed, Jan 8</td>
<td>Logical Equivalence/Gates</td>
<td>1.1-1.3</td>
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<td>Fri, Jan 10</td>
<td>More Logic/Circuits</td>
<td>1.2-2.2</td>
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<td>Set Theory</td>
<td>2.1-2.3</td>
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<td>Modular Arithmetic</td>
<td>4.1-4.2</td>
<td>3.4-3.5</td>
<td>6th</td>
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<td>12</td>
<td>Fri, Feb 3</td>
<td>Applications of Mod, Number Theory, Factoring</td>
<td>4.1-3.3</td>
<td>3.4-3.6</td>
<td>6th</td>
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<td>13</td>
<td>Mon, Feb 5</td>
<td>GCD, Euclid's Algorithm, Modular Equations</td>
<td>4.3-4.4</td>
<td>3.4-3.7</td>
<td>6th</td>
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<td>14</td>
<td>Wed, Feb 7</td>
<td>Induction</td>
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<td>4.1</td>
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<td>Fri, Feb 7</td>
<td>More Induction</td>
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<tr>
<th>name</th>
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<th>Room</th>
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<tr>
<td>Siddharth Iyer</td>
<td>Th 11:30-12:30</td>
<td>DEN 213</td>
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<tr>
<td>Suraj Jagadeesh</td>
<td>Th 12:30-1:30</td>
<td>LOW 220</td>
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<tr>
<td>Karthuma Manthik</td>
<td>Th 1:30-2:30</td>
<td>LOW 101</td>
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<td>Josh Shin</td>
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<td>Xiaoyu Sun</td>
<td>Th 2:30-3:20</td>
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<tr>
<td>Jason Wataja</td>
<td>Th 1:30-2:30</td>
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Section Materials

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<thead>
<tr>
<th>Date</th>
<th>Problems</th>
<th>Solns</th>
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<td>Jan 9</td>
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Homework

Exams:
- Final exam: The final exam will be at the officially scheduled time, Monday 16-Mar-2020, 2:30-4:20 pm.
Work

Homework:
Due WED at 11:00 pm online (Gradescope)
Write up individually
Extra Credit

Exams:
Midterm in class on Friday, Feb 14
Final exam:
  Monday, March 16  2:30-4:20 pm

Grading (roughly):
  50% Homework
  15-20% Midterm
  30-35% Final Exam
Communication

• You are already on the class e-mail list
  – Major announcements here, archive reachable from the course webpage

• If you want to email to us (me & TAs):
  cse311-staff@cs.washington.edu

• Discussion board
  – accept invitation to Ed class discussion board
About grades...

• Grades were very important up until now...
About grades...

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• Grades are much less important going forward
  – companies care much more about your interviews
  – grad schools care much more about recommendations
About grades...

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• Understanding the material is much more important  
  – interviews test your knowledge from these classes  
  – good recommendations involve knowledge beyond the classes
About grades...

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• Understanding the material is much more important
  – interviews test your knowledge from these classes
  – good recommendations involve knowledge beyond the classes

• Please relax and focus on learning
Please calm down about grades

• Most time spent on questions about grading issues is not worthwhile to either the student or teacher

• Try to avoid asking “will I lose points if...”

• If the thought of losing points worries you, show more work
  – no sense having a 30 minute discussion to save 10 minutes
Collaboration Policy

• Collaboration with others is encouraged
• BUT you must:
  – list anyone you work with
  – turn in only your own work

• Recommended approach for group work
  – do not leave with any solution written down or photographed
  – wait 30 minutes before writing up your solution

• See Allen School Academic Misconduct policy also
No Late Days

- To be accepted, late submission (with good reason) must be arranged in advance 48 hours before the deadline
If you are worried about Mathy aspects of 311

• Associated 1-credit CR/NC workshop
  – CSE 390ZA (not yet available for enrollment)
  – Extra collaborative practice on 311 concepts, study skills, a small amount of assigned work
  – 1.5 hours Thursdays 3:30 pm
  – Full attendance is required, else NC
  – NOT for help with 311 homework

• Anyone in 311 can sign up but enrollment is limited
Getting used to being formal

As problems we deal with get harder we need stronger tools...

Formalism is a tool we apply when problems get difficult
  – helps us get through without making mistakes
  – sometimes even gives “turn the crank” solutions
Propositional Logic
What is logic and why do we need it?

Logic is a language, like English or Java, with its own
• words and rules for combining words into sentences (syntax)
• ways to assign meaning to words and sentences (semantics)

Why learn another language when we know English and Java already?
Why not use English?

– Turn right here...

– Buffalo buffalo Buffalo buffalo buffalo buffalo buffalo buffalo Buffalo buffalo

– We saw her duck
Why not use English?

– Turn right here...
  Does “right” mean the direction or now?

– Buffalo buffalo Buffalo buffalo buffalo buffalo buffalo Buffalo buffalo buffalo
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

– We saw her duck
  Does “duck” mean the animal or crouch down?
Why not use English?

– Turn right here...
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– Buffalo buffalo Buffalo buffalo buffalo
  buffalo Buffalo buffalo
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

– We saw her duck
  Does “duck” mean the animal or crouch down?

Natural languages can be imprecise
Why not use Java?

What does this code do:

```java
public static boolean mystery(int x) {
    for (int r = 2; r < x; r++) {
        for (int q = 2; q < x; q++) {
            if (r*q == x)
                return false;
        }
    }
    return x > 1;
}
```
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Determines if x is a prime number
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        }
    }
    return x > 1;
}
```

Determines if x is a prime number

Programming languages can be verbose
Why learn a new language?

We need a language of reasoning to

– state sentences more precisely
– state sentences more concisely
– understand sentences more quickly
A *proposition* is a statement that
- is either true or false
- is “well-formed”
Propositions: building blocks of logic

A *proposition* is a statement that

– is either true or false

– is “well-formed”

All cats are mammals

true

All mammals are cats

false
Are These Propositions?

2 + 2 = 5 \[\text{Y}\]

x + 2 = 5 \[\text{N}\]

Akjsdf! \[\text{N}\]

Who are you? \[\text{N}\]

Every positive even integer can be written as the sum of two primes. \[\text{Y}\]
Are These Propositions?

2 + 2 = 5
   This is a proposition. It’s okay for propositions to be false.

x + 2 = 5
   Not a proposition. Doesn’t have a fixed truth value

Akjsdf!
   Not a proposition because it’s gibberish.

Who are you?
   This is a question which means it doesn’t have a truth value.

Every positive even integer can be written as the sum of two primes.
   This is a proposition. We don’t know if it’s true or false, but we know it’s one of them!
A first application of logic

“If I were to ask you out, would your answer to that question be the same as your answer to this one?”
We need a way of talking about *arbitrary* ideas...

Propositional Variables: $p, q, r, s, \ldots$

Truth Values:
- $T$ for *true*
- $F$ for *false*
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to understand what this proposition means.
A Compound Proposition

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to understand what this proposition means.

First find the simplest (atomic) propositions:

\( p \)  “Garfield has black stripes”

\( q \)  “Garfield is an orange cat”

\( r \)  “Garfield likes lasagna”
“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

We’d like to understand what this proposition means.

First find the simplest (atomic) propositions:

- $p$ “Garfield has black stripes”
- $q$ “Garfield is an orange cat”
- $r$ “Garfield likes lasagna”

$\underbrace{(p \text{ if } (q \text{ and } r)) \text{ and } (q \text{ or } \text{not } r)}$
<table>
<thead>
<tr>
<th>Logical Connectives</th>
<th>Symbol</th>
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<tbody>
<tr>
<td>Negation (not)</td>
<td>$\neg p$</td>
</tr>
<tr>
<td>Conjunction (and)</td>
<td>$p \land q$</td>
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<tr>
<td>Disjunction (or)</td>
<td>$p \lor q$</td>
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<tr>
<td>Exclusive Or</td>
<td>$p \oplus q$</td>
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<tr>
<td>Implication</td>
<td>$p \rightarrow q$</td>
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<tr>
<td>Biconditional</td>
<td>$p \leftrightarrow q$</td>
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## Logical Connectives

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<th>Negation (not)</th>
<th>( \neg p )</th>
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<td>( p \rightarrow q )</td>
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<tr>
<td>Biconditional</td>
<td>( p \iff q )</td>
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</table>

**“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”**

\[
(p \text{ if } (q \text{ and } r)) \text{ and } (q \text{ or } (\neg r))
\]
Logical Connectives

Negation (not) \( \neg p \)

Conjunction (and) \( p \land q \)

Disjunction (or) \( p \lor q \)

Exclusive Or \( p \oplus q \)

Implication \( p \rightarrow q \)

Biconditional \( p \iff q \)

“(Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

\( (p \text{ if } (q \land r)) \land (q \lor \neg r) \)
Some Truth Tables

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## Some Truth Tables

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Implication

“If it’s raining, then I have my umbrella”

It’s useful to think of implications as promises. That is “Did I lie?”

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<tr>
<th></th>
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<table>
<thead>
<tr>
<th></th>
<th>It’s raining</th>
<th>It’s not raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have my umbrella</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>I do not have my umbrella</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>
Implication

“If it’s raining, then I have my umbrella”

It’s useful to think of implications as promises. That is “Did I lie?”

The only lie is when:
(a) It’s raining AND
(b) I don’t have my umbrella
Implication

“If it’s raining, then I have my umbrella”

Are these true?

\[
\begin{array}{ccc}
p & q & p \rightarrow q \\
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\end{array}
\]

\[
2 + 2 = 4 \rightarrow \text{earth is a planet}
\]

\[
2 + 2 = 5 \rightarrow 26 \text{ is prime}
\]
Implication

“If it’s raining, then I have my umbrella”

Are these true?

\[ 2 + 2 = 4 \rightarrow \text{earth is a planet} \]

The fact that these are unrelated doesn’t make the statement false! “2 + 2 = 4” is true; “earth is a planet” is true. \( T \rightarrow T \) is true. So, the statement is true.

\[ 2 + 2 = 5 \rightarrow 26 \text{ is prime} \]

Again, these statements may or may not be related. “2 + 2 = 5” is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!
(1) “I have collected all 151 Pokémon if I am a Pokémon master”
(2) “I have collected all 151 Pokémon only if I am a Pokémon master”

These sentences are implications in opposite directions:
These sentences are implications in opposite directions:
(1) “Pokémon masters have all 151 Pokémon”
(2) “People who have 151 Pokémon are Pokémon masters”

So, the implications are:
(1) *If* I am a Pokémon master, *then* I have collected all 151 Pokémon.
(2) *If* I have collected all 151 Pokémon, *then* I am a Pokémon master.
$p \rightarrow q$

Implication:
- $p$ implies $q$
- whenever $p$ is true $q$ must be true
- if $p$ then $q$
- $q$ if $p$
- $p$ is sufficient for $q$
- $p$ only if $q$
- $q$ is necessary for $p$

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Biconditional: $p \leftrightarrow q$

- $p$ iff $q$
- $p$ is equivalent to $q$
- $p$ implies $q$ and $q$ implies $p$
- $p$ is necessary and sufficient for $q$

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Biconditional: $p \iff q$

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Back to Garfield...

\[ p \quad \text{“Garfield has black stripes”} \]
\[ q \quad \text{“Garfield is an orange cat”} \]
\[ r \quad \text{“Garfield likes lasagna”} \]

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

\[ (p \text{ if } (q \land r)) \land (q \lor \neg r) \]

\[ (p \text{ “if” } (q \land r)) \land (q \lor \neg r) \]
Back to Garfield...

$p$ “Garfield has black stripes”
$q$ “Garfield is an orange cat”
$r$ “Garfield likes lasagna”

“Garfield has black stripes if he is an orange cat and likes lasagna, and he is an orange cat or does not like lasagna”

$(p \text{ if } (q \text{ and } r)) \text{ and } (q \text{ or } \lnot r)$

$(p \text{ “if” } (q \land r)) \land (q \lor \lnot r)$

$((q \land r) \rightarrow p) \land (q \lor \lnot r)$
Analyzing the Garfield Sentence with a Truth Table

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