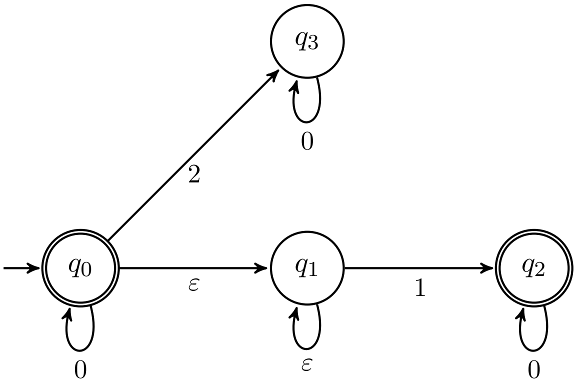
CSE 311: Foundations of Computing I

Section 9: Minimization, NFAs, Subset Construction Solutions

1. NFAs

(a) What language does the following NFA accept?



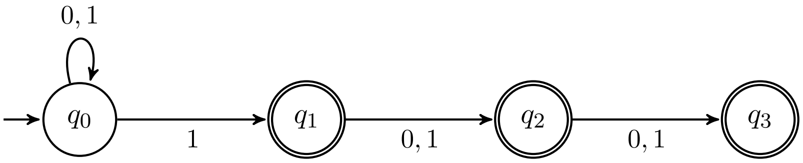
Solution:

All strings of only 0's and 1's not containing more than one 1.

(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

Solution:

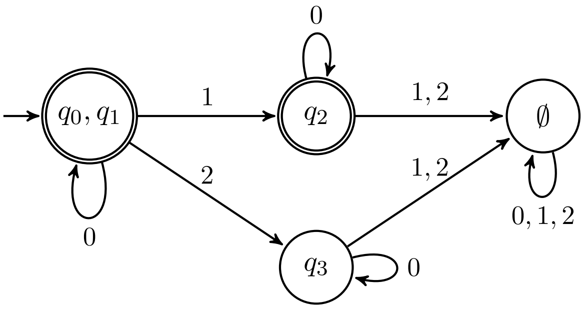
The following is one such NFA:



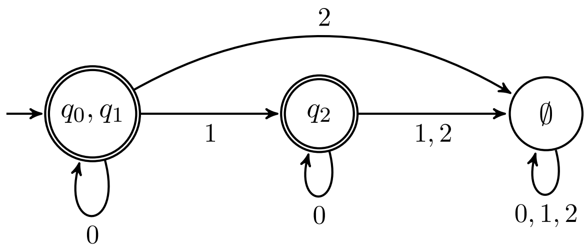
2. DFAs & Minimization

(a) Convert the NFA from 1a to a DFA, then minimize it.

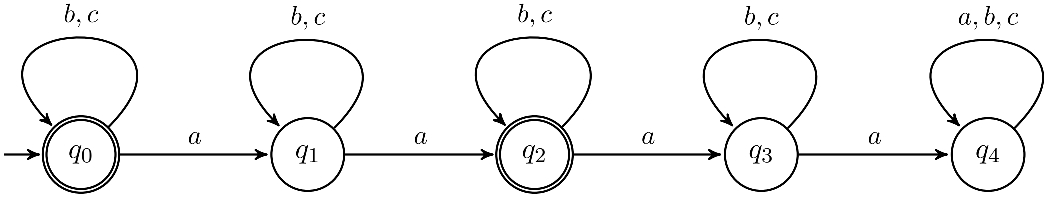
Solution:



Here is the minimized form:



(b) Minimize the following DFA:



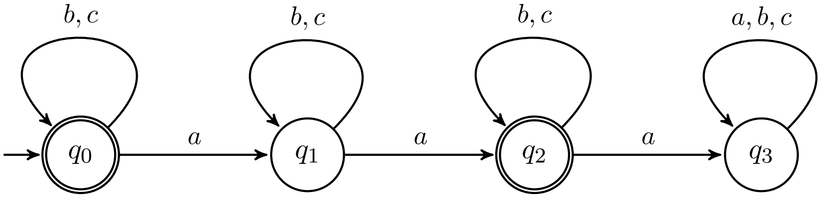
Solution:

Step 1: q_0, q_2 are final states and the rest are not final. So, we start with the initial partition with the following groups: group 1 is $\{q_0, q_2\}$ and group 2 is $\{q_1, q_3, q_4\}$.

Step 2: q_1 is sending a to group 1 while q_3, q_4 are sending a to group 2. So, we divide group 2. We get the following groups: group 1 is $\{q_0, q_2\}$, group 3 is $\{q_1\}$ and group 4 is $\{q_3, q_4\}$.

Step 3: q_0 is sending a to group 3 and q_2 is sending a to group 4. So, we divide group 1. We will have the following groups: group 3 is $\{q_1\}$, group 4 is $\{q_3, q_4\}$, group 5 is $\{q_0\}$ and group 6 is $\{q_2\}$.

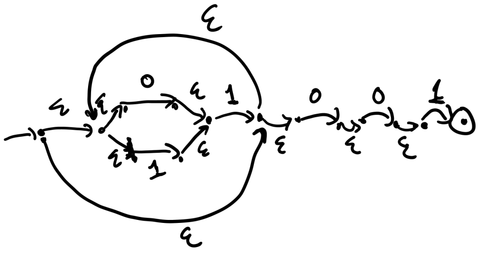
The minimized DFA is the following:

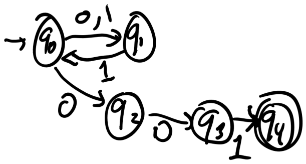


3. RegExp to NFA

Use our generic construction to build an NFA that recognizes the language given by the following regular expression: $((0 \cup 1)1)^*001$. If you have time, also give as small an NFA as you can. (Unlike with DFAs there is no good minimization algorithm known for NFAs.)

Solution:


ε
ε
ε
0
ε
1
ε
ε
1
ε
0
0
1
ε
ε
ε
ε


q_0
0,1
q_1
1
0
q_2
0
q_3
1
q_4