CSE 311: Foundations of Computing I

Section 7: Structural Induction and Regular Expressions

1. Strong Induction repeat question

Xavier Cantelli owns some rabbits. The number of rabbits he has in any given year is described by the function f:

$$f(0) = 0$$
$$f(1) = 1$$
$$f(n) = 2f(n-1) - f(n-2) \text{ for } n \geq 2$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year n.

2. Structural Induction

(a) Consider the following recursive definition of strings.

Basis Step: "" is a string

Recursive Step: If X is a string and c is a character then $\mathsf{append}(c, X)$ is a string.

Recall the following recursive definition of the function len:

$$\mathsf{len}(\text{""}) = 0$$
$$\mathsf{len}(\mathsf{append}(c, X)) = 1 + \mathsf{len}(X)$$

Now, consider the following recursive definition:

$$\mathsf{double}(\text{""}) = \text{""}$$
$$\mathsf{double}(\mathsf{append}(c, X)) = \mathsf{append}(c, \mathsf{append}(c, \mathsf{double}(X))).$$

Prove that for any string X, $\mathsf{len}(\mathsf{double}(X)) = 2\mathsf{len}(X)$.

(b) Consider the following definition of a (binary) **Tree**:

Basis Step: $\bullet$ is a **Tree**.

Recursive Step: If L is a **Tree** and R is a **Tree** then $\mathtt{Tree}(\bullet, L, R)$ is a **Tree**.

The function leaves returns the number of leaves of a **Tree**. It is defined as follows:

$$\mathsf{leaves}(\bullet) = 1$$
$$\mathsf{leaves}(\mathtt{Tree}(\bullet, L, R)) = \mathsf{leaves}(L) + \mathsf{leaves}(R)$$

Also, recall the definition of size on trees:

$$\mathsf{size}(\bullet) = 1$$
$$\mathsf{size}(\mathtt{Tree}(\bullet, L, R)) = 1 + \mathsf{size}(L) + \mathsf{size}(R)$$

Prove that $\mathsf{leaves}(T) \geq \mathsf{size}(T)/2 + 1/2$ for all Trees T.

3. Regular Expressions

(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3.

(c) Write a regular expression that matches all binary strings that contain the substring "111", but not the substring "000".