# CSE 311: Foundations of Computing I

## Section 6: Induction and Strong Induction

### 1. Induction

(a) Prove that $9 \mid (n^3 + (n+1)^3 + (n+2)^3)$ for all $n > 1$ by induction.

(b) Prove that $6n + 6 < 2^n$ for all $n \geq 6$.

(c) Define

$$H_i = 1 + \frac{1}{2} + \cdots + \frac{1}{i}$$

Prove that $H_{2^n} \geq 1 + \frac{n}{2}$ for $n \in \mathbb{N}$.

### 2. Strong Induction

Xavier Cantelli owns some rabbits. The number of rabbits he has in any given year is described by the function $f$:

$$f(0) = 0$$
$$f(1) = 1$$
$$f(n) = 2f(n-1) - f(n-2) \text{ for } n \geq 2$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year $n$.