CSE 311: Foundations of Computing I

Section 3: Inference Solutions

1. Formal Proof (Direct Proof Rule)

Show that $\neg p \rightarrow s$ follows from $p \vee q$, $q \rightarrow r$ and $r \rightarrow s$.

Solution:

1.	$p \vee q$	[Given]
2.	$q \rightarrow r$	[Given]
3.	$r \rightarrow s$	[Given]
4.1.	$\neg p$	[Assumption]
4.2.	q	[Elim of $\vee$: 1, 4.1]
4.3.	r	[MP of 4.2, 2]
4.4.	s	[MP 4.3, 3]
4.	$\neg p \rightarrow s$	[Direct Proof Rule]

2. Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \rightarrow q) \wedge (r \rightarrow s)$.

Solution:

1.	$\neg(\neg r \vee t)$	[Given]
2.	$\neg q \vee \neg s$	[Given]
3.	$(p \rightarrow q) \wedge (r \rightarrow s)$	[Given]
4.	$\neg\neg r \wedge \neg t$	[DeMorgan's Law: 1]
5.	$\neg\neg r$	[Elim of $\wedge$: 4]
6.	r	[Double Negation: 5]
7.	$r \rightarrow s$	[Elim of $\wedge$: 3]
8.	s	[MP, 6,7]
9.	$\neg\neg s$	[Double Negation: 8]
10.	$\neg s \vee \neg q$	[Commutative: 2]
11.	$\neg q$	[Elim of $\vee$: 10, 9]
12.	$p \rightarrow q$	[Elim of $\wedge$: 3]
13.	$\neg q \rightarrow \neg p$	[Contrapositive: 12]
14.	$\neg p$	[MP: 11,13]

3. A Formal Proof in Predicate Logic

Prove $\exists x\ (P(x) \vee R(x))$ from $\forall x\ (P(x) \vee Q(x))$ and $\forall y\ (\neg Q(y) \vee R(y))$.

Solution:

1.	$\forall x\ (P(x) \vee Q(x))$	[Given]
2.	$\forall y\ (\neg Q(y) \vee R(y))$	[Given]
3.	$P(a) \vee Q(a)$	[Elim ∀: 1]
4.	$\neg Q(a) \vee R(a)$	[Elim ∀: 2]
5.	$Q(a) \to R(a)$	[Law of Implication: 4]
6.	$\neg\neg P(a) \vee Q(a)$	[Double Negation: 3]
7.	$\neg P(a) \to Q(a)$	[Law of Implication: 5]
	8.1. $\neg P(a)$ [Assumption]	
	8.2. $Q(a)$ [Modus Ponens: 8.1, 7]	
	8.3. $R(a)$ [Modus Ponens: 8.2, 5]	
8.	$\neg P(a) \to R(a)$	[Direct Proof]
9.	$\neg\neg P(a) \vee R(a)$	[Law of Implication: 8]
10.	$P(a) \vee R(a)$	[Double Negation: 9]
11.	$\exists x\ (P(x) \vee R(x))$	[Intro ∃: 10]