CSE 311: Foundations of Computing I

Section 1: Logic Solutions

1. Exclusive Or

For each of the following, decide whether inclusive-or or exclusive-or is intended:

(a) Experience with C or Java is required.

Solution:

Inclusive Or.

(b) Lunch includes soup or salad.

Solution:

Exclusive Or.

(c) Publish or perish

Solution:

Exclusive Or.

(d) To enter the country you need a passport or Global Entry card.

Solution:

Inclusive Or.

2. Translations

For each of the following, define propositional variables and translate the sentences into logical notation.

(a) I will remember to send you the address only if you send me an e-mail message.

Solution:

p : I will remember to send you the address

q : You send me an e-mail message

$$\boxed{p \rightarrow q}$$

(b) If berries are ripe along the trail, hiking is safe if and only if grizzly bears have not been seen in the area.

Solution:

p : Berries are ripe along the trail

q : Hiking is safe

r : Grizzly bears have not been seen in the area

$$\boxed{p \rightarrow (q \leftrightarrow r)}$$

(c) Unless I am trying to type something, my cat is either eating or sleeping.

Solution:

$$p : \text{My cat is eating}$$
$$q : \text{My cat is sleeping}$$
$$r : \text{I'm trying to type}$$

$$\boxed{\neg r \to (p \oplus q)}$$

3. Teatime

Consider the following sentence:

If I am drinking tea then I am eating a cookie, or, if I am eating a cookie then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation.

Solution:

$$p : \text{I am drinking tea}$$
$$q : \text{I am eating a cookie}$$

$$\boxed{(p \to q) \vee (q \to p)}$$

(b) Fill out a truth table for your expression.

Solution:

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \vee (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

(c) Based on your truth table, classify the original sentence as a contingency, tautology, or contradiction.

Solution:

Tautology

4. Truth Tables

Write a truth table for each of the following:

(a) $(p \oplus q) \vee (p \oplus \neg q)$

Solution:

p	q	$p \oplus q$	$p \oplus \neg q$	$(p \oplus q) \vee (p \oplus \neg q)$
T	T	F	T	T
T	F	T	F	T
F	T	T	F	T
F	F	F	T	T

(b) $(p \vee q) \rightarrow (p \oplus q)$

Solution:

p	q	$p \vee q$	$p \oplus q$	$(p \vee q) \rightarrow (p \oplus q)$
T	T	T	F	F
T	F	T	T	T
F	T	T	T	T
F	F	F	F	T

(c) $p \leftrightarrow \neg p$

Solution:

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

5. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a) $p \rightarrow q$ $\qquad\qquad$ $q \rightarrow p$

Solution:

When $p =$ T and $q =$ F, then $p \rightarrow q \equiv$ F, but $q \rightarrow p \equiv$ T.

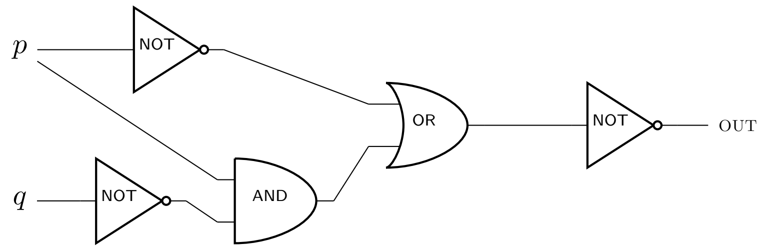
(b) $p \rightarrow (q \wedge r)$ $\qquad\qquad$ $(p \rightarrow q) \wedge r$

Solution:

When $p =$ F and $r =$ F, then $p \rightarrow (q \wedge r) \equiv$ T, but $(p \rightarrow q) \wedge r \equiv$ F.

6. Circuitous

Translate the following circuit into a logical expression.



Solution:

$\neg(\neg p \lor (p \land \neg q))$