CSE 311: Foundations of Computing I

Homework 3 (due Wednesday, January 29th at 11:00 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture, the theorems handout, and previous homeworks without proof. Read the CSE 311 grading guidelines from the course webpage for more details and for permitted resources and collaboration.*

1. Translating into logic (12 points)

Translate the following sentences into predicate logic. The domain of discourse is the set of all positive integers. You can use the predicates: Even(x), Odd(x), Prime(x), Greater(x, y), Equal(x, y), Sum(x, y, z) from Lecture 5 and 6.

(a) [3 Points] Every prime number is the sum of two primes.

(b) [3 Points] For all positive integers x, there are two prime numbers larger than x that differ by 2.

(c) [3 Points] There is prime number that is smaller than all even numbers.

(d) [3 Points] There are no even prime numbers.

2. Not So Negative (15 points)

For each of the following English statements, (i) translate it into predicate logic, (ii) write the negation of that statement in predicate logic with the negation symbols pushed as far in as possible so that any negation symbols is directly in front of a predicate, and then (iii) translate the result of (ii) back to English (natural if possible).

For the logic, let your domain of discourse be people and activities. You should use only the predicates Loves(x, y), Likes(x, y), and Hates(x, y), which say that person x loves, likes, or hates (respectively) activity y; the predicates Person(x) and Activity(x), which say whether x is a person or activity (respectively); and the predicate Equal(x, y), which says whether x and y are the same object.

(a) [5 Points] Fred likes some activity other than hiking.

(b) [5 Points] There is someone who doesn't love any activity but likes every activity.

(c) [5 Points] Everyone who likes hiking and swimming has an activity that they love.

3. Formal Proofs (24 points)

(a) [14 Points] Write a formal proof using inference rules that given $(p \wedge \neg q) \vee (\neg p \wedge q)$, $r \rightarrow \neg s$, and $(s \wedge p) \rightarrow r$, the proposition $s \rightarrow q$ must also be true.

(b) [10 Points] Write a formal proof using inference rules of $((p \rightarrow q) \wedge (r \rightarrow \neg q)) \rightarrow (r \rightarrow \neg p)$

4. Spoofclusions (17 points)

Theorem: Given $s \to (p \wedge q)$, $\neg s \to r$, and $(r \vee p) \to q$, prove q.

"Spoof:"

1.	$\neg s \to r$	Given
2.	$(r \vee p) \to q$	Given
3.	$r \to q$	$\vee$ Elim: 2
	4.1. $\neg s$ Assumption	
	4.2. r MP: 4.1, 1	
	4.3. q MP: 4.2, 3	
4.	$\neg s \to q$	Direct Proof Rule
	5.1. s Assumption	
	5.2. $s \to (p \wedge q)$ Given	
	5.3. $p \wedge q$ MP: 5.1, 5.2	
	5.4. q $\wedge$ Elim: 5.3	
5.	$s \to q$	[Direct Proof Rule]
6.	$(s \to q) \wedge (\neg s \to q)$	$\wedge$ Intro: 4, 5
7.	$(\neg s \vee q) \wedge (\neg\neg s \vee q)$	Law of Implication
8.	$(\neg s \vee q) \wedge (s \vee q)$	Double Negation
9.	$((\neg s \vee q) \wedge s) \vee ((\neg s \vee q) \wedge q)$	Distributivity
10.	$((\neg s \vee q) \wedge s) \vee (q \wedge (\neg s \vee q))$	Commutativity
11.	$((\neg s \vee q) \wedge s) \vee (q \wedge (q \vee \neg s))$	Commutativity
12.	$((\neg s \vee q) \wedge s) \vee q$	Absorption
13.	$(s \wedge (\neg s \vee q)) \vee q$	Commutativity
14.	$((s \wedge \neg s) \vee q) \vee q$	Associativity
15.	$(F \vee q) \vee q$	Negation
16.	$(q \vee F) \vee q$	Commutativity
17.	$q \vee q$	Identity
18.	q	Idempotence

(a) [6 Points] There are two errors in this proof. Indicate which lines contain the errors and, for each one, explain (as briefly as possible) why that line is incorrect.

(b) [5 Points] Is the conclusion of the "spoof" correct? Explain why or why not.

(c) [6 Points] Give a correct proof of what is claimed in lines 6–18, i.e., that, from $(s \to q) \wedge (\neg s \to q)$, we can infer that q is true.

5. Mind Your P's and Q's (18 points)

Using the logical inference rules and equivalences we have given, write a *formal* proof that given $\forall x\ (\exists y\ P(x,y) \to \neg Q(x))$, $\forall x\ (\neg R(x) \to (Q(x) \vee \neg P(x,x))$, and $\exists x\ P(x,x)$, you can conclude that $\exists x\ R(x)$.

6. Hip to be square (14 points)

Give a formal proof that, if integers n and m are squares, then nm is a square. In addition to the inference rules discussed in class, you can also rewrite an algebraic expression to equivalent ones using the rule "Algebra".

7. Extra credit: Compared to what? (0 points)

In this problem, you will design a circuit with a minimal number of gates that takes a pair of length four bit strings $x_3x_2x_1x_0$ and $y_3y_2y_1y_0$ and returns a single bit indicating whether the binary integers they represent, $(x_3x_2x_1x_0)_2$ and $(y_3y_2y_1y_0)_2$, satisfy $(x_3x_2x_1x_0)_2 < (y_3y_2y_1y_0)_2$. See the following table for some examples.

$x_3x_2x_1x_0$	$y_3y_2y_1y_0$	$(x_3x_2x_1x_0)_2 < (y_3y_2y_1y_0)_2$
0101	1011	1
1100	0111	0
1101	1101	0

Design such a circuit using at most 10 AND, OR, and XOR gates. You can use an arbitrary number of NOT gates, and a single gate can have multiple inputs. (Extra credit points start at 10 gates, but if you can use fewer, you will get even more points.)