CSE 311: Foundations of Computing I

Homework 2 (due Wednesday, January 22 at 11:00 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. You may use results from lecture, the theorems handout, and previous homeworks without proof. Read the CSE 311 grading guidelines from the course webpage for more details and for permitted resources and collaboration.*

Below, for proofs in Propositional Logic, you must cite every rule that you apply, including *commutativity* and *associativity*. You may apply only a single rule per line. However, you can apply that rule to multiple parts of the formula as long as they are non-overlapping.

For proofs in Boolean Algebra, however, you may also skip citing *associativity* throughout your answer. In fact, because associativity is normal in the ordinary algebra we are used to, you may skip the parentheses altogether when their only purpose is to indicate the order of application for two identical operators, e.g. $(X+Y)+Z$ vs. $X+(Y+Z)$ or $(XY)X$ vs $X(YZ)$.

1. It's all the same to me... (18 points)

Prove the following assertions using equivalences. You can use commutativity and associativity an arbitrary number of times in a single line of the proof.

(a) [6 Points]

$$p \to (q \to r) \equiv ((p \land q) \to r).$$

(b) [12 Points]

$$((p \to r) \land (q \to \neg r)) \to (\neg p \lor \neg q) \equiv T.$$

(For this problem, for simplicity of presentation you can also use the "Left Distributive" laws, which say that $(p \lor q) \land r \equiv (p \land r) \lor (q \land r)$ and $(p \land q) \lor r \equiv (p \lor r) \land (q \lor r)$. These can be derived by using Commutativity once then ordinary Distributivity followed by two parallel applications of Commutativity.)

2. This is BAD (Boolean Algebra Deduction)! (10 points)

Show that the Boolean Algebra expression $((X'+Y)(Z'+Y'))'+Z'+X'$ can be simplified to the constant 1.

3. Counting Courses with Combinational Logic (20 points)

In lecture 4, we considered a combinational logic example about days of class.

(a) [5 Points] Write c_1 in the product of sum form.

(b) [6 Points] Simplify the sum of product forms of c_0, c_2 using boolean algebra axioms and theorems. Make sure to cite which axioms and theorems you are using when simplifying.

(c) [9 Points] Simplify the sum of product forms of c_1 using boolean algebra axioms and theorems.

4. A Steady Diet (8 points)

Suppose that our domain of discourse is all menu items. Consider the following predicates:

- Let $B(x)$ be "x is available for breakfast."
- Let $P(x)$ be "x has lots of protein ."

Translate each of the following into English.

(a) [2 Points] $\exists x(B(x) \land P(x))$

(b) [2 Points] $\forall x(B(x) \to P(x))$

(c) [2 Points] $\forall x(B(x) \land P(x))$

(d) [2 Points] $\exists x(B(x) \to P(x))$

5. Pardon My French (16 points)

Let the domain of discourse be people in France. Let's define the predicates Francophone(x) and Anglophone(x) to mean that x is a French speaker or an English speaker, respectively. Define the predicates French(x), Parisian(x), and Tourist(x) to mean that x is French, lives in Paris, or is a tourist, respectively.

Translate each of the following logical statements into English. You should not simplify. However, you should use the techniques shown in lecture for producing more natural translations when restricting domains and for avoiding the introduction of variable names when not necessary.

(a) [4 Points] $\forall x\,(\text{Parisian}(x) \to (\text{French}(x) \lor \text{Tourist}(x)))$

(b) [4 Points] $\exists x\,(\text{Anglophone}(x) \land \text{Tourist}(x) \land \neg\,\text{Francophone}(x))$

(c) [4 Points] $\Big(\forall x\,((\text{Parisian}(x) \land \neg\,\text{Tourist}(x)) \to \text{French}(x))\Big) \land \neg \Big(\forall x\,(\,\text{French}(x) \to (\text{Parisian}(x) \lor \text{Tourist}(x)))\Big)$

(d) [4 Points] $\neg\exists x\,\Big(\big(\,\text{Tourist}(x) \land \neg(\text{Francophone}(x) \lor \text{Anglophone}(x))\big) \land \text{Parisian}(x)\Big)$

6. Book Shelf Bingo (16 points)

For this problem, let the domain of discourse be UW CSE faculty and the books on their office shelves. We will also define the following predicates:

- Let Equal(x, y) be "x = y" — either the same faculty member or the same book.
- Let Same(x, y) be "x and y are books with the same content".
- Let OnShelf(x, y) be "book x is on the book shelf in y's office."

Translate each of the following English statements into logic using only the quantifiers $\exists$ and $\forall$ as defined in class.

(a) [4 Points] Two faculty members have books with the same content on their shelves.

(b) [4 Points] No faculty member has two books with the same content on their shelf.

(c) [4 Points] Book x is *not* the only book with its content.

(d) [4 Points] Some faculty member has a book on their shelf that is the only one (anywhere) with its content.

7. Beyond Compare (12 points)

The questions below consider the two propositions

$$\forall x\,(P(x) \to Q(x)) \quad \text{and} \quad (\forall x\, P(x)) \to (\forall x\, Q(x))$$

where P and Q are predicates.

(a) [6 Points] Give examples of predicates P and Q and a domain of discourse so that the two propositions are **not** equivalent.

(b) [6 Points] Give examples of predicates P and Q and a domain of discourse so that they **are** equivalent.

(c) [0 Points] **Extra credit**: What logical relationship holds between these two propositions? Explain.

8. Extra Credit: Aarh Me Hearties! (0 points)

Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins. On their ship, they decide to split the coins using the following scheme:

- The first pirate in alphabetical order becomes the chief pirate.
- The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way.
- Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain.

Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent (and aware that all the other pirates are just as aware, intelligent, and bloodthirsty), what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.