

## Section 10: Irregularity, Cardinality, Uncountability

### 1. Irregularity

- (a) Let  $\Sigma = \{0, 1\}$ . Prove that  $\{0^n 1^n 0^n : n \geq 0\}$  is not regular.
- (b) Let  $\Sigma = \{0, 1, 2\}$ . Prove that  $\{0^n (12)^m : n \geq m \geq 0\}$  is not regular.

### 2. Cardinality

- (a) You are a pirate. You begin in a square on a 2D grid which is infinite in all directions. In other words, wherever you are, you may move up, down, left, or right. Some single square on the infinite grid has treasure on it. Find a way to ensure you find the treasure in finitely many moves.
- (b) Prove that  $\{3x : x \in \mathbb{N}\}$  is countable.
- (c) Prove that the set of irrational numbers is uncountable.  
**Hint:** Use the fact that the rationals are countable and that the reals are uncountable.
- (d) Prove that  $\mathcal{P}(\mathbb{N})$  is uncountable.

### 3. Uncomputability

- (a) Let  $\Sigma = \{0, 1\}$ . Prove that the set of palindromes is decidable.
- (b) Prove that the set  $\{(\text{CODE}(R), x, y) : R \text{ is a program and } R(x) \neq R(y)\}$  is undecidable where we write  $R(x) = \uparrow$  if  $R$  runs forever.