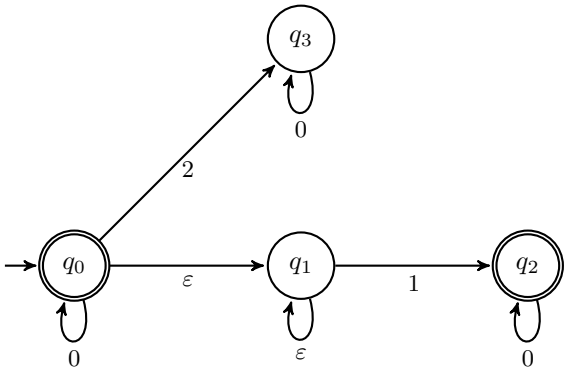
Section 9: NFAs, Subset Construction, and Review

1. NFAs

(a) Recall the following NFA from last section. What language does the NFA accept?



(b) Convert the NFA form part (a) to a DFA.

2. Translate to Logic

Express each of these system specifications using predicates, quantifiers, and logical connectives.

(a) Every user has access to an electronic mailbox.

(b) The system mailbox can be accessed by everyone in the group if the file system is locked.

(c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.

(d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

3. Palindromes

We say an integer is *palindromic* if the digits read the same when written forward or backward. Prove that every palindromic integer with an even number of digits is divisible by 11. (No induction proofs.)

Hint 1: $10 \equiv -1 (\text{mod } 11)$.
Hint 2: Write the number in terms of its $2n$ decimal digits as $d_0 + d_1 \cdot 10 + d_2 \cdot 10^2 + \cdots d_{2n-1} \cdot 10^{2n-1}$

4. Multiplicative Inverses

For p a prime number, show that for all $n \in [p-1]$, there exists a unique multiplicative inverse of n mod p. In other words for all $n \in [p-1]$, there exists a unique $m \in [p-1]$ so that $n \cdot m \equiv 1 \pmod{p}$.

5. Polygonal chords

A polygon is a 2 dimensional shape made of straight line segments with at least 3 vertices. We define a chord of a polygon to be a straight line joining two non-adjacent vertices of the polygon. A convex polygon is a polygon such that any chord lies in its interior. What is the maximum number of non-intersecting chords a convex polygon on n vertices can have?

The insight for this problem is challenging! If you don't get it after some thought, be sure to look at the solution.