Section 3: Inference

1. Using the Direct Proof Rule

Show that $\neg p \to s$ follows from $p \vee q$, $q \to r$ and $r \to s$.

2. A Formal Proof in Propositional Logic

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \to q) \wedge (r \to s)$.

3. A Formal Proof in Predicate Logic

Prove $\exists x\ (P(x) \vee R(x))$ from $\forall x\ (P(x) \vee Q(x))$ and $\forall y\ (\neg Q(y) \vee R(y))$.

4. Formal Spoofs

For each of of the following proofs, determine why the proof is incorrect. Then, show that the claim is true by fixing the error.

(a) Show that $p \to (q \vee r)$ follows from $p \to q$ and r.

1.	$p \to q$	[Given]
2.	r	[Given]
3.	$p \to (q \vee r)$	[$\vee$ Intro: 1, 2]

(b) Show that q follows from $\neg p \vee q$ and p.

1.	$\neg p \vee q$	[Given]
2.	p	[Given]
3.	q	[$\vee$ Elim: 1, 2]

(c) Show that q follows from $q \vee p$ and $\neg p$.

1.	$\neg p$	[Given]
2.	$q \vee p$	[Given]
3.	$q \vee \mathsf{F}$	[Substitute $p = \mathsf{F}$ since $\neg p$ holds: 1, 2]
4.	q	[Identity: 3]