Section 3: Inference

1. Using the Direct Proof Rule
Show that \( \neg p \rightarrow s \) follows from \( p \lor q \), \( q \rightarrow r \) and \( r \rightarrow s \).

2. A Formal Proof in Propositional Logic
Show that \( \neg p \) follows from \( \neg (\neg r \lor t) \), \( \neg q \lor \neg s \) and \( (p \rightarrow q) \land (r \rightarrow s) \).

3. A Formal Proof in Predicate Logic
Prove \( \exists x \ (P(x) \lor R(x)) \) from \( \forall x \ (P(x) \lor Q(x)) \) and \( \forall y \ (\neg Q(y) \lor R(y)) \).

4. Formal Spoofs
For each of the following proofs, determine why the proof is incorrect. Then, show that the claim is true by fixing the error.

(a) Show that \( p \rightarrow (q \lor r) \) follows from \( p \rightarrow q \) and \( r \).

1. \( p \rightarrow q \) [Given]
2. \( r \) [Given]
3. \( p \rightarrow (q \lor r) \) [\lor Intro: 1, 2]

(b) Show that \( q \) follows from \( \neg p \lor q \) and \( p \).

1. \( \neg p \lor q \) [Given]
2. \( p \) [Given]
3. \( q \) [\lor Elim: 1, 2]

(c) Show that \( q \) follows from \( q \lor p \) and \( \neg p \).

1. \( \neg p \) [Given]
2. \( q \lor p \) [Given]
3. \( q \lor F \) [Substitute \( p = F \) since \( \neg p \) holds: 1, 2]
4. \( q \) [Identity: 3]