# Section 1: Logic

## 1. Exclusive Or

For each of the following, decide whether inclusive-or or exclusive-or is intended:

(a) Experience with C or Java is required.

**Solution:**

Inclusive Or.

(b) Lunch includes soup or salad.

**Solution:**

Exclusive Or.

(c) Publish or perish.

**Solution:**

Exclusive Or.

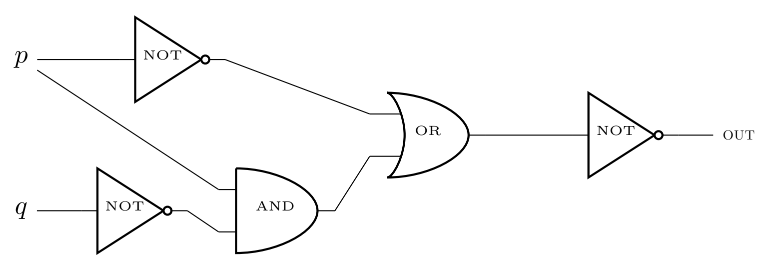
(d) To enter the country you need a passport or voter registration card.

**Solution:**

Inclusive Or.

## 2. Circuitous

Translate the following circuit into a logical expression.




**Solution:**

$\neg(\neg p \vee (p \wedge \neg q))$

## 3. Translations

For each of the following, define propositional variables and translate the sentences into logical notation.

(a) If berries are ripe along the trail, hiking is safe and grizzly bears have not been seen in the area.

**Solution:**

$p$ : Berries are ripe along the trail
$q$ : Hiking is safe
$r$ : Grizzly bears have been seen in the area

$$\boxed{p \rightarrow (q \wedge \neg r)}$$

(b) Unless I am trying to type something, my cat is either eating or sleeping.

**Solution:**

$$p : \text{My cat is eating}$$
$$q : \text{My cat is sleeping}$$
$$r : \text{I'm trying to type}$$

"If" makes a claim only when the premise is true. "Unless" make a claim only when the proposition following it is *not true*, so it usually translates as "if not". In this case, that gives us:

$$\boxed{\neg r \to (p \oplus q)}$$

(c) If it's on sale, I'll buy canned cat food. Otherwise, I'll buy dry cat food.

**Solution:**

$$p : \text{Canned cat food is on sale}$$
$$q : \text{I'll buy canned cat food}$$
$$r : \text{I'll buy dry cat food}$$

$$\boxed{(p \to q) \wedge (\neg p \to r)}$$

## 4. Truth Tables

Write a truth table for each of the following:

(a) $p \to \mathsf{T}$

**Solution:**

| $p$ | T | $p \to \mathsf{T}$ |
|---|---|---|
| T | T | T |
| F | T | T |

(b) $\mathsf{F} \to p$

**Solution:**

| F | $p$ | $\mathsf{F} \to p$ |
|---|---|---|
| F | T | T |
| F | F | T |

(c) Explain, in English, what the truth tables for (a) and (b) tell us?

**Solution:**

(a) says that anything (whether true or false) implies true. (b) says that false implies anything.

(d) $(p \oplus q) \vee (p \oplus \neg q)$

**Solution:**

| $p$ | $q$ | $p \oplus q$ | $p \oplus \neg q$ | $(p \oplus q) \vee (p \oplus \neg q)$ |
|---|---|---|---|---|
| T | T | F | T | T |
| T | F | T | F | T |
| F | T | T | F | T |
| F | F | F | T | T |

(e) $(p \vee q) \rightarrow (p \oplus q)$

**Solution:**

| $p$ | $q$ | $p \vee q$ | $p \oplus q$ | $(p \vee q) \rightarrow (p \oplus q)$ |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | T | T | T |
| F | T | T | T | T |
| F | F | F | F | T |

## 5. Teatime

Consider the following sentence:

If I am drinking tea, then I am eating a cookie, or, if I am eating a cookie, then I am drinking tea.

(a) Define propositional variables and translate the sentence into an expression in logical notation.

**Solution:**

$p$ : I am drinking tea
$q$ : I am eating a cookie

$$\boxed{(p \rightarrow q) \vee (q \rightarrow p)}$$

(b) Fill out a truth table for your expression.

**Solution:**

| $p$ | $q$ | $(p \rightarrow q)$ | $(q \rightarrow p)$ | $(p \rightarrow q) \vee (q \rightarrow p)$ |
|---|---|---|---|---|
| T | T | T | T | T |
| T | F | F | T | T |
| F | T | T | F | T |
| F | F | T | T | T |

(c) Based on your truth table, classify the original sentence as a contingency, tautology, or contradiction.

**Solution:**

Tautology.

## 6. Interview Question

The following is an old interview question:

*There are three boxes, one contains only apples, one contains only oranges, and one contains both apples and oranges. The boxes have been incorrectly labeled such that no label identifies the actual contents of its box.*

*Opening just one box, and without looking in the box, you take out one piece of fruit. By looking at the fruit, how can you immediately label all of the boxes correctly?*

(a) Create a table showing all the logical possibilities for what could be in each of the boxes. Then indicate which possibilities are consistent with the description in the problem. To simplify the table, you need only list those possibilities where each fruit combination (apples, oranges, or both) appears in exactly one box.

**Solution:**

| Apples | Oranges | Both | claim |
|---|---|---|---|
| apples | oranges | both | F |
| apples | both | oranges | F |
| oranges | apples | both | F |
| oranges | both | apples | T |
| both | apples | oranges | T |
| both | oranges | apples | F |

(b) If you take a fruit from the box labelled "Apples", will you always know what is in the other boxes? Why or why not?

**Solution:**

No. If you happen to get an apple, then you know they must be arranged as in the fifth row, not the fourth row. But if you pick an orange, that is consistent with both the fourth and fifth rows, so you cannot tell which situation you are in.

(c) How do you solve the problem?

**Solution:**

Pick a fruit from the box labelled "Both". If it is an apple, we are in the fourth row, and if it is an orange, we are in the fifth row.