CSE 311 Reference Sheets: Key Definitions and Results

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Topics

Logical equivalences
List of basic equivalences in propositional logic.

Boolean algebra identities
List of basic identities in boolean algebra.

Inference rules
List of inference rules for propositional and predicate logic.

Set theory definitions
List of definitions related to set theory.

Logical equivalences

List of basic equivalences in propositional logic.

Basic equivalences in propositional logic

DeMorgan's laws

$$\neg(p \land q) \equiv \neg p \lor \neg q$$
$$\neg(p \lor q) \equiv \neg p \land \neg q$$

Law of implication

$$p \to q \equiv \neg p \lor q$$

Contrapositive

$$p \to q \equiv \neg q \to \neg p$$

Biconditional

$$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$$

Double negation

$$p \equiv \neg\neg p$$

Identity

$$p \land \mathsf{T} \equiv p$$
$$p \lor \mathsf{F} \equiv p$$

Domination

$$p \land \mathsf{F} \equiv \mathsf{F}$$
$$p \lor \mathsf{T} \equiv \mathsf{T}$$

Idempotence

$$p \land p \equiv p$$
$$p \lor p \equiv p$$

Commutativity

$$p \land q \equiv q \land p$$
$$p \lor q \equiv q \lor p$$

Associativity

$$(p \land q) \land r \equiv p \land (q \land r)$$
$$(p \lor q) \lor r \equiv p \lor (q \lor r)$$

Distributivity

$$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$$
$$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$$

Absorption

$$p \land (p \lor q) \equiv p$$
$$p \lor (p \land q) \equiv p$$

Negation

$$p \land \neg p \equiv \mathsf{F}$$
$$p \lor \neg p \equiv \mathsf{T}$$

Boolean algebra identities

List of basic identities in boolean algebra.

Boolean algebra axioms and (some) theorems

Closure

$$a + b \in B$$
$$a \cdot b \in B$$

Commutativity

$$a + b = b + a$$
$$a \cdot b = b \cdot a$$

Associativity

$$a + (b + c) = (a + b) + c$$
$$a \cdot (b \cdot c) = (a \cdot b) \cdot c$$

Distributivity

$$a + (b \cdot c) = (a + b) \cdot (a + c)$$
$$a \cdot (b + c) = (a \cdot b) + (a \cdot c)$$

Identity

$$a + 0 = a$$
$$a \cdot 1 = a$$

Complementarity

$$a + a' = 1$$
$$a \cdot a' = 0$$

Null

$$a + 1 = 1$$
$$a \cdot 0 = 0$$

Idempotency

$$a + a = a$$
$$a \cdot a = a$$

Involution

$$(a')' = a$$

Uniting

$$a \cdot b + a \cdot b' = a$$
$$(a + b) \cdot (a + b') = a$$

Absorption

$$a + a \cdot b = a$$
$$a \cdot (a + b) = a$$
$$(a + b') \cdot b = a \cdot b$$
$$(a \cdot b') + b = a + b$$

Factoring

$$(a + b) \cdot (a' + c) = a \cdot c + a' \cdot b$$
$$a \cdot b + a' \cdot c = (a + c) \cdot (a' + b)$$

Consensus

$$(a \cdot b) + (b \cdot c) + (a' \cdot c) = a \cdot b + a' \cdot c$$
$$(a + b) \cdot (b + c) \cdot (a' + c) = (a + b) \cdot (a' + c)$$

DeMorgan's

$$(a + b + \ldots)' = a' \cdot b' \cdot \ldots$$
$$(a \cdot b \cdot \ldots)' = a' + b' + \ldots$$

Inference rules

List of inference rules for propositional and predicate logic.

Inference rules for propositional and predicate logic

Intro ∧ $\dfrac{A; B}{\therefore A \wedge B}$

Elim ∧ $\dfrac{A \wedge B}{\therefore A, B}$

Intro ∨ $\dfrac{A}{\therefore A \vee B, B \vee A}$

Elim ∨ $\dfrac{A \vee B; \neg A}{\therefore B}$

Direct Proof Rule $\dfrac{A \implies B}{\therefore A \rightarrow B}$

Modus Ponens $\dfrac{A; A \rightarrow B}{\therefore B}$

Excluded Middle $\dfrac{}{\therefore A \vee \neg A}$

Elim ∀ $\dfrac{\forall x.\, P(x)}{\therefore P(a) \text{ for any } a}$

Intro ∀ $\dfrac{P(a); a \text{ is arbitrary}}{\therefore \forall x.\, P(x)}$

The name a stands for an arbitrary value in the domain. No other name in P depends on a.

Intro ∃ $\dfrac{P(c) \text{ for some } c}{\therefore \exists x.\, P(x)}$

Elim ∃ $\dfrac{\exists x.\, P(x)}{\therefore P(c) \text{ for a specific } c}$

The name c is **fresh** and stands for a value in the domain where $P(c)$ is true. List all dependencies for c.

Set theory definitions

List of definitions related to set theory.

Set definitions and operations

Common sets

$\mathbb{N} = \{0, 1, 2, \dots\}$ is the set of natural numbers.
$\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$ is the set of integers.
$\mathbb{Q} = \{\frac{p}{q} : p, q \in \mathbb{Z} \wedge q \neq 0\}$ is the set of rational numbers.
$\mathbb{R}$ is the set of real numbers.

Containment, equality, and subsets

$x \in A$ means that x is an element of A.
$x \notin A$ means that x is *not* an element of A.
$A \subseteq B$ means that A is a subset of B: $\forall x.\, x \in A \rightarrow x \in B$.
$A = B$ means that A and B have the same elements: $\forall x.\, x \in A \leftrightarrow x \in B$.

Set operations

$A \cup B = \{x : (x \in A) \vee (x \in B)\}$ is the union of A and B.
$A \cap B = \{x : (x \in A) \wedge (x \in B)\}$ is the intersection of A and B.
$A \setminus B = \{x : (x \in A) \wedge (x \notin B)\}$ is the difference of A and B.
$A \oplus B = \{x : (x \in A) \oplus (x \in B)\}$ is the symmetric difference of A and B.
$\overline{A} = \{x : x \notin A\} = \{x : \neg(x \in A)\}$ is the complement of A with respect to a universe U.

Set constructions

$S = \{x : P(x)\}$ means that S is the set of all x in the domain of P for which $P(x)$ is true.
$A \times B = \{(a, b) : a \in A \wedge b \in B\}$ is the cartesian product of A and B.
$\mathcal{P}(A) = \{B : B \subseteq A\}$ is the power set of A.
$[n] = \{1, 2, \dots, n\}$ is the set of natural numbers from 1 to n.