CSE 311 Lecture 03: Equivalence and Proofs

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Topics

Equivalence and circuits

A brief review of Lecture 02.

Checking equivalence

Applications and a basic brute-force algorithm.

Logical proofs

A method for establishing equivalence that extends to richer logics.

Equivalence and circuits

A brief review of Lecture 02.

Logical equivalence

$A \equiv B$ is an **assertion** that two propositions A and B have the same truth values in all possible cases.

$A \equiv B$ and $\underbrace{(A \leftrightarrow B) \equiv \mathsf{T}}_{\text{tautology}}$ have the same meaning.

$$p \wedge q \equiv q \wedge p$$

p	q	$p \wedge q$	$q \wedge p$	$p \wedge q \leftrightarrow q \wedge p$
F	F	F	F	T
F	T	F	F	T
T	F	F	F	T
T	T	T	T	T

$$p \wedge q \not\equiv q \vee p$$

When $p = \mathsf{T}$ and $q = \mathsf{F}$, $p \wedge q$ is false but $p \vee q$ is true!

Important equivalences

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

We will always give you this list!

Digital circuits

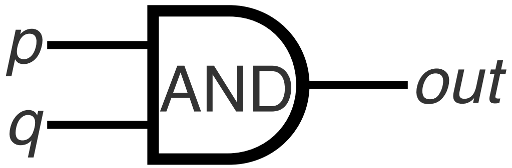
Digital circuits implement propositional logic:

- T corresponds to 1 or high voltage.
- F corresponds to 0 or low voltage.

Digital gates are functions that

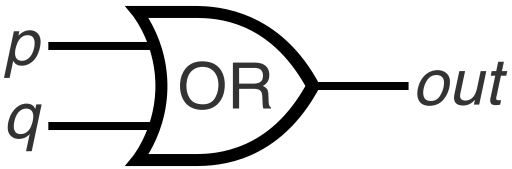
- take values 0/1 as inputs and produce 0/1 as output;
- correspond to logical connectives (many of them).

AND, OR, and NOT gates



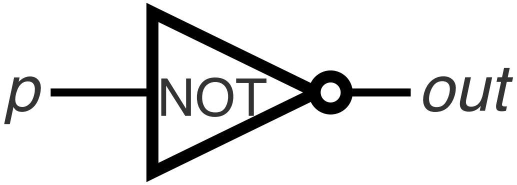
p	q	out
0	0	0
0	1	0
1	0	0
1	1	1

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T



p	q	out
0	0	0
0	1	1
1	0	1
1	1	1

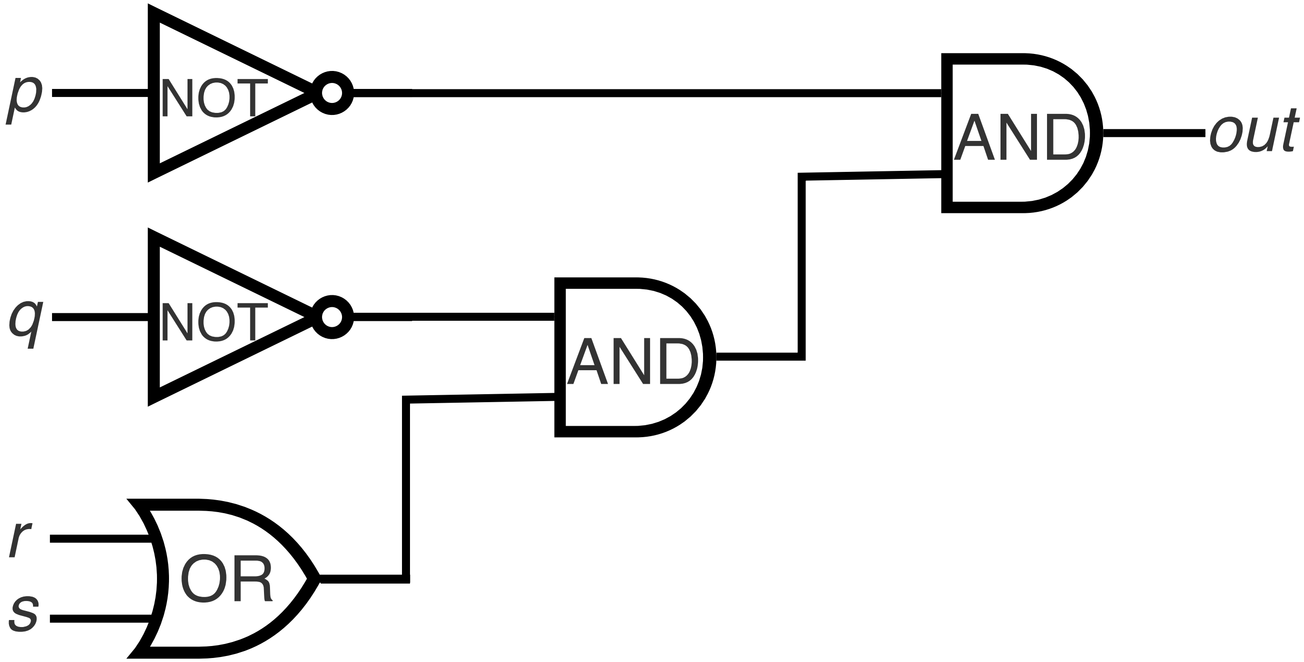
p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T



p	out
0	1
1	0

p	$\neg p$
F	T
T	F

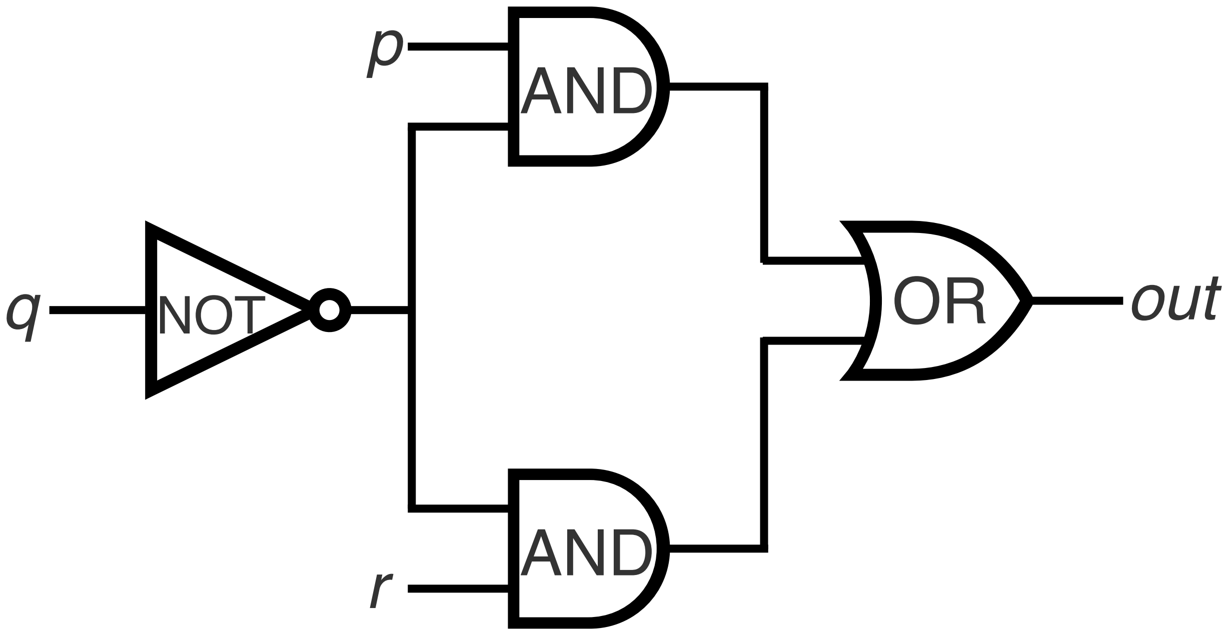
Combinational logic circuits: wiring up gates



Values get sent along wires connecting gates.

$\neg p \wedge (\neg q \wedge (r \vee s))$

Combinational logic circuits: wiring up gates



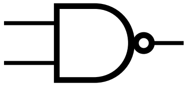
Wires can send one value to multiple gates.

$(p \wedge \neg q) \vee (\neg q \wedge r)$

Other useful gates

NAND gate

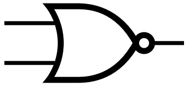
$\neg(p \land q)$



p	q	out
0	0	1
0	1	1
1	0	1
1	1	0

NOR gate

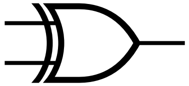
$\neg(p \lor q)$



p	q	out
0	0	1
0	1	0
1	0	0
1	1	0

XOR gate

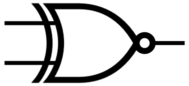
$p \oplus q$



p	q	out
0	0	0
0	1	1
1	0	1
1	1	0

XNOR gate

$p \leftrightarrow q$



p	q	out
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Checking equivalence

Applications and a basic brute-force algorithm.

Why do we care about checking equivalence?

Many practical problems are solved by logical equivalence checking!

Hardware verification, program verification, query optimization and caching, compiler optimization, …

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Example: verifying compiler optimizations

Given a sequence of instructions S and an optimized sequence P, we can construct logical formulas s and p that encode their meaning. To verify that P behaves exactly like S, we check that $p \leftrightarrow s \equiv \top$.

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Demo: verifying compiler peephole optimizations with Alive

Is this optimization correct?

```
; Original program (S)
%a = xor %y, %x ; a = y ^ x
%b = and %y, %x ; b = x & x
%c = ashr %b, 1 ; c = b >> 1
%d = add %c, %a ; d = c + a
  =>
; Optimized program (P)
%d = add %x, %y ; d = x + y
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%d = add %c, %a ; d = c + a
  =>
; Optimized program (P)
%d = add %x, %y ; d = x + y
```

No! The right shift (`ashr`) should be replaced with a left shift (`shl`).

Checking logical (and circuit) equivalence

Can we write an algorithm to decide if two propositions are equivalent?

What is the run time of the algorithm?

In theory, the news are bad …

But in practice, the news are pretty good …

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But in practice, the news are pretty good …

Provers like Z3 can solve equivalence checking problems with millions of variables and formulas. And that's enough for many real applications!

Logical proofs

A method for establishing equivalence that extends to richer logics.

Proof: use known equivalences to derive new ones

To show that A is equivalent to B

Apply a series of logical equivalences to subexpressions to convert A to B.

To show that A is a tautology

Apply a series of logical equivalences to subexpressions to convert A to $\top$.

Example: show that A is equivalent to B

Let A be $p \lor (p \land p)$, and let B be p.

$$
\begin{aligned}
p \lor (p \land p) &\equiv \\
&\equiv p
\end{aligned}
$$

DeMorgan's laws

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) \equiv \neg p \land \neg q$

Law of implication

$p \to q \equiv \neg p \lor q$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \land \mathsf{T} \equiv p$

$p \lor \mathsf{F} \equiv p$

Domination

$p \land \mathsf{F} \equiv \mathsf{F}$

$p \lor \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \land p \equiv p$

$p \lor p \equiv p$

Commutativity

$p \land q \equiv q \land p$

$p \lor q \equiv q \lor p$

Associativity

$(p \land q) \land r \equiv p \land (q \land r)$

$(p \lor q) \lor r \equiv p \lor (q \lor r)$

Distributivity

$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Absorption

$p \land (p \lor q) \equiv p$

$p \lor (p \land q) \equiv p$

Negation

$p \land \neg p \equiv \mathsf{F}$

$p \lor \neg p \equiv \mathsf{T}$

Example: show that A is equivalent to B

Let A be $p \vee (p \wedge p)$, and let B be p.

$$\begin{aligned} p \vee (p \wedge p) &\equiv p \vee p && \text{Idempotence} \\ &\equiv p \end{aligned}$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

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Absorption

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Negation

$p \wedge \neg p \equiv \mathsf{F}$

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Example: show that A is equivalent to B

Let A be $p \lor (p \land p)$, and let B be p.

$$
\begin{aligned}
p \lor (p \land p) &\equiv p \lor p && \text{Idempotence} \\
&\equiv p && \text{Idempotence}
\end{aligned}
$$

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Commutativity

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Associativity

$(p \land q) \land r \equiv p \land (q \land r)$

$(p \lor q) \lor r \equiv p \lor (q \lor r)$

Distributivity

$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Absorption

$p \land (p \lor q) \equiv p$

$p \lor (p \land q) \equiv p$

Negation

$p \land \neg p \equiv \mathsf{F}$

$p \lor \neg p \equiv \mathsf{T}$

Example: show that A is a tautology

Let A be $\neg p \vee (p \vee p)$.

$$\begin{aligned} \neg p \vee (p \vee p) &\equiv \\ &\equiv \\ &\equiv \mathsf{T} \end{aligned}$$

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$\neg(p \wedge q) \equiv \neg p \vee \neg q$

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Distributivity

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Absorption

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Negation

$p \wedge \neg p \equiv \mathsf{F}$

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Let A be $\neg p \lor (p \lor p)$.

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Negation

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Example: show equivalence with a truth table and proof

$p \land (p \to q) \equiv p \land q$

A **truth table** for $p \land (p \to q) \leftrightarrow p \land q \equiv$ T.

p	q	$p \to q$	$p \land (p \to q)$	$p \land q$	$p \land (p \to q) \leftrightarrow p \land q$
F	F	T	F	F	T
F	T	T	F	F	T
T	F	F	F	F	T
T	T	T	T	T	T

Example: show equivalence with a truth table and proof

$$p \wedge (p \rightarrow q) \equiv p \wedge q$$

$$\begin{aligned} p \wedge (p \rightarrow q) &\equiv \\ &\equiv \\ &\equiv \\ &\equiv \\ &\equiv p \wedge q \end{aligned}$$

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Example: show equivalence with a truth table and proof

$$p \wedge (p \to q) \equiv p \wedge q$$

$$\begin{aligned} p \wedge (p \to q) &\equiv p \wedge (\neg p \vee q) \\ &\equiv \\ &\equiv \\ &\equiv \\ &\equiv p \wedge q \end{aligned}$$

Law of implication

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$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \to q \equiv \neg p \vee q$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show equivalence with a truth table and proof

$$p \wedge (p \rightarrow q) \equiv p \wedge q$$

$$\begin{aligned}
p \wedge (p \rightarrow q) &\equiv p \wedge (\neg p \vee q) && \text{Law of implication} \\
&\equiv (p \wedge \neg p) \vee (p \wedge q) && \text{Distributivity} \\
&\equiv \\
&\equiv \\
&\equiv p \wedge q
\end{aligned}$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show equivalence with a truth table and proof

$$p \wedge (p \to q) \equiv p \wedge q$$

$$\begin{aligned} p \wedge (p \to q) &\equiv p \wedge (\neg p \vee q) && \text{Law of implication} \\ &\equiv (p \wedge \neg p) \vee (p \wedge q) && \text{Distributivity} \\ &\equiv \mathsf{F} \vee (p \wedge q) && \text{Negation} \\ &\equiv \\ &\equiv p \wedge q \end{aligned}$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \to q \equiv \neg p \vee q$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show equivalence with a truth table and proof

$$p \wedge (p \rightarrow q) \equiv p \wedge q$$

$$\begin{aligned} p \wedge (p \rightarrow q) &\equiv p \wedge (\neg p \vee q) && \text{Law of implication} \\ &\equiv (p \wedge \neg p) \vee (p \wedge q) && \text{Distributivity} \\ &\equiv \mathsf{F} \vee (p \wedge q) && \text{Negation} \\ &\equiv (p \wedge q) \vee \mathsf{F} && \text{Commutativity} \\ &\equiv p \wedge q \end{aligned}$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show equivalence with a truth table and proof

$p \wedge (p \rightarrow q) \equiv p \wedge q$

$p \wedge (p \rightarrow q)$	$\equiv p \wedge (\neg p \vee q)$	Law of implication
	$\equiv (p \wedge \neg p) \vee (p \wedge q)$	Distributivity
	$\equiv \mathsf{F} \vee (p \wedge q)$	Negation
	$\equiv (p \wedge q) \vee \mathsf{F}$	Commutativity
	$\equiv p \wedge q$	Identity

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \wedge q) \rightarrow (q \vee p)$

A **truth table** for $(p \wedge q) \rightarrow (q \vee p) \equiv \mathsf{T}$.

p	q	$p \wedge q$	$q \vee p$	$(p \wedge q) \rightarrow (q \vee p)$
F	F	F	F	T
F	T	F	T	T
T	F	F	T	T
T	T	T	T	T

Example: show tautology with a truth table and proof

$$(p \land q) \to (q \lor p)$$

$$
\begin{aligned}
(p \land q) \to (q \lor p) &\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) \equiv \neg p \land \neg q$

Law of implication

$p \to q \equiv \neg p \lor q$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \land \mathsf{T} \equiv p$

$p \lor \mathsf{F} \equiv p$

Domination

$p \land \mathsf{F} \equiv \mathsf{F}$

$p \lor \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \land p \equiv p$

$p \lor p \equiv p$

Commutativity

$p \land q \equiv q \land p$

$p \lor q \equiv q \lor p$

Associativity

$(p \land q) \land r \equiv p \land (q \land r)$

$(p \lor q) \lor r \equiv p \lor (q \lor r)$

Distributivity

$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Absorption

$p \land (p \lor q) \equiv p$

$p \lor (p \land q) \equiv p$

Negation

$p \land \neg p \equiv \mathsf{F}$

$p \lor \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \wedge q) \rightarrow (q \vee p)$

$$
\begin{aligned}
(p \wedge q) \rightarrow (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) && \text{Law of implication} \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \wedge q) \rightarrow (q \vee p)$

$$
\begin{aligned}
(p \wedge q) \rightarrow (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) && \text{Law of implication} \\
&\equiv (\neg p \vee \neg q) \vee (q \vee p) && \text{DeMorgan} \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \wedge q) \rightarrow (q \vee p)$

$$
\begin{aligned}
(p \wedge q) \rightarrow (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) && \text{Law of implication} \\
&\equiv (\neg p \vee \neg q) \vee (q \vee p) && \text{DeMorgan} \\
&\equiv \neg p \vee (\neg q \vee (q \vee p)) && \text{Associativity} \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$$(p \wedge q) \to (q \vee p)$$

$$\begin{aligned}
(p \wedge q) \to (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) && \text{Law of implication} \\
&\equiv (\neg p \vee \neg q) \vee (q \vee p) && \text{DeMorgan} \\
&\equiv \neg p \vee (\neg q \vee (q \vee p)) && \text{Associativity} \\
&\equiv \neg p \vee ((\neg q \vee q) \vee p) && \text{Associativity} \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \to q \equiv \neg p \vee q$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \wedge q) \rightarrow (q \vee p)$

$$
\begin{aligned}
(p \wedge q) \rightarrow (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) && \text{Law of implication} \\
&\equiv (\neg p \vee \neg q) \vee (q \vee p) && \text{DeMorgan} \\
&\equiv \neg p \vee (\neg q \vee (q \vee p)) && \text{Associativity} \\
&\equiv \neg p \vee ((\neg q \vee q) \vee p) && \text{Associativity} \\
&\equiv \neg p \vee (p \vee (\neg q \vee q)) && \text{Commutativity} \\
&\equiv \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \wedge q) \rightarrow (q \vee p)$

$$
\begin{aligned}
(p \wedge q) \rightarrow (q \vee p) &\equiv \neg(p \wedge q) \vee (q \vee p) && \text{Law of implication} \\
&\equiv (\neg p \vee \neg q) \vee (q \vee p) && \text{DeMorgan} \\
&\equiv \neg p \vee (\neg q \vee (q \vee p)) && \text{Associativity} \\
&\equiv \neg p \vee ((\neg q \vee q) \vee p) && \text{Associativity} \\
&\equiv \neg p \vee (p \vee (\neg q \vee q)) && \text{Commutativity} \\
&\equiv (\neg p \vee p) \vee (\neg q \vee q) && \text{Associativity} \\
&\equiv \\
&\equiv \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \vee q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \wedge \mathsf{T} \equiv p$

$p \vee \mathsf{F} \equiv p$

Domination

$p \wedge \mathsf{F} \equiv \mathsf{F}$

$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \wedge p \equiv p$

$p \vee p \equiv p$

Commutativity

$p \wedge q \equiv q \wedge p$

$p \vee q \equiv q \vee p$

Associativity

$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption

$p \wedge (p \vee q) \equiv p$

$p \vee (p \wedge q) \equiv p$

Negation

$p \wedge \neg p \equiv \mathsf{F}$

$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$$(p \wedge q) \rightarrow (q \vee p)$$

$(p \wedge q) \rightarrow (q \vee p) \equiv \neg(p \wedge q) \vee (q \vee p)$	Law of implication
$\equiv (\neg p \vee \neg q) \vee (q \vee p)$	DeMorgan
$\equiv \neg p \vee (\neg q \vee (q \vee p))$	Associativity
$\equiv \neg p \vee ((\neg q \vee q) \vee p)$	Associativity
$\equiv \neg p \vee (p \vee (\neg q \vee q))$	Commutativity
$\equiv (\neg p \vee p) \vee (\neg q \vee q)$	Associativity
$\equiv (p \vee \neg p) \vee (q \vee \neg q)$	Commutativity (twice)
$\equiv$	
$\equiv \mathsf{T}$	

DeMorgan's laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$
$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Law of implication
$p \rightarrow q \equiv \neg p \vee q$

Contrapositive
$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional
$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Double negation
$p \equiv \neg\neg p$

Identity
$p \wedge \mathsf{T} \equiv p$
$p \vee \mathsf{F} \equiv p$

Domination
$p \wedge \mathsf{F} \equiv \mathsf{F}$
$p \vee \mathsf{T} \equiv \mathsf{T}$

Idempotence
$p \wedge p \equiv p$
$p \vee p \equiv p$

Commutativity
$p \wedge q \equiv q \wedge p$
$p \vee q \equiv q \vee p$

Associativity
$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$
$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Distributivity
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption
$p \wedge (p \vee q) \equiv p$
$p \vee (p \wedge q) \equiv p$

Negation
$p \wedge \neg p \equiv \mathsf{F}$
$p \vee \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$$(p \land q) \to (q \lor p)$$

$$
\begin{aligned}
(p \land q) \to (q \lor p) &\equiv \neg(p \land q) \lor (q \lor p) && \text{Law of implication} \\
&\equiv (\neg p \lor \neg q) \lor (q \lor p) && \text{DeMorgan} \\
&\equiv \neg p \lor (\neg q \lor (q \lor p)) && \text{Associativity} \\
&\equiv \neg p \lor ((\neg q \lor q) \lor p) && \text{Associativity} \\
&\equiv \neg p \lor (p \lor (\neg q \lor q)) && \text{Commutativity} \\
&\equiv (\neg p \lor p) \lor (\neg q \lor q) && \text{Associativity} \\
&\equiv (p \lor \neg p) \lor (q \lor \neg q) && \text{Commutativity (twice)} \\
&\equiv \mathsf{T} \lor \mathsf{T} && \text{Negation (twice)} \\
&\equiv \mathsf{T}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) \equiv \neg p \land \neg q$

Law of implication

$p \to q \equiv \neg p \lor q$

Contrapositive

$p \to q \equiv \neg q \to \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \to q) \land (q \to p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \land \mathsf{T} \equiv p$

$p \lor \mathsf{F} \equiv p$

Domination

$p \land \mathsf{F} \equiv \mathsf{F}$

$p \lor \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \land p \equiv p$

$p \lor p \equiv p$

Commutativity

$p \land q \equiv q \land p$

$p \lor q \equiv q \lor p$

Associativity

$(p \land q) \land r \equiv p \land (q \land r)$

$(p \lor q) \lor r \equiv p \lor (q \lor r)$

Distributivity

$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Absorption

$p \land (p \lor q) \equiv p$

$p \lor (p \land q) \equiv p$

Negation

$p \land \neg p \equiv \mathsf{F}$

$p \lor \neg p \equiv \mathsf{T}$

Example: show tautology with a truth table and proof

$(p \land q) \rightarrow (q \lor p)$

$$
\begin{aligned}
(p \land q) \rightarrow (q \lor p) &\equiv \neg(p \land q) \lor (q \lor p) && \text{Law of implication}\\
&\equiv (\neg p \lor \neg q) \lor (q \lor p) && \text{DeMorgan}\\
&\equiv \neg p \lor (\neg q \lor (q \lor p)) && \text{Associativity}\\
&\equiv \neg p \lor ((\neg q \lor q) \lor p) && \text{Associativity}\\
&\equiv \neg p \lor (p \lor (\neg q \lor q)) && \text{Commutativity}\\
&\equiv (\neg p \lor p) \lor (\neg q \lor q) && \text{Associativity}\\
&\equiv (p \lor \neg p) \lor (q \lor \neg q) && \text{Commutativity (twice)}\\
&\equiv \mathsf{T} \lor \mathsf{T} && \text{Negation (twice)}\\
&\equiv \mathsf{T} && \text{Idempotence}
\end{aligned}
$$

DeMorgan's laws

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) \equiv \neg p \land \neg q$

Law of implication

$p \rightarrow q \equiv \neg p \lor q$

Contrapositive

$p \rightarrow q \equiv \neg q \rightarrow \neg p$

Biconditional

$p \leftrightarrow q \equiv (p \rightarrow q) \land (q \rightarrow p)$

Double negation

$p \equiv \neg\neg p$

Identity

$p \land \mathsf{T} \equiv p$

$p \lor \mathsf{F} \equiv p$

Domination

$p \land \mathsf{F} \equiv \mathsf{F}$

$p \lor \mathsf{T} \equiv \mathsf{T}$

Idempotence

$p \land p \equiv p$

$p \lor p \equiv p$

Commutativity

$p \land q \equiv q \land p$

$p \lor q \equiv q \lor p$

Associativity

$(p \land q) \land r \equiv p \land (q \land r)$

$(p \lor q) \lor r \equiv p \lor (q \lor r)$

Distributivity

$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

Absorption

$p \land (p \lor q) \equiv p$

$p \lor (p \land q) \equiv p$

Negation

$p \land \neg p \equiv \mathsf{F}$

$p \lor \neg p \equiv \mathsf{T}$

Truth tables versus proofs

Proofs are not smaller than truth tables where there are a few propositional variables.

But proofs are usually much smaller when there are many variables.

We can extend the proof method to reason about richer logics for which truth tables don't apply.

Theorem provers use a combination of search (truth tables) and deduction (proofs) to automate equivalence checking.

Summary

Checking equivalence has many real-world applications.

Verification, optimization, and more!

There are two ways to check equivalence of propositional formulas.

Brute-force: compare their truth tables.

Proof-based: apply equivalences to transform one into the other.