# CSE 311 Lecture 01: Propositional Logic

Emina Torlak and Sami Davies

# Topics

**About CSE 311**

What you will learn and why!

**Course logistics**

A quick summary now; details on the course webpage.

**Propositional logic**

A language of reasoning

# About CSE 311

What you will learn and why!

# Learn the calculus of computation

**Logic**

How do we describe ideas precisely?

**Formal proofs**

How can we be sure we’re right?

**Number theory**

How do we keep data secure?

**Relations**

How do we organize information?

**Finite state machines**

How do we design hardware and software?

**Turing machines**

Are there problems computers can’t solve?

# And become a better programmer!

By the end of the course, you will have the key technical tools to …

- reason about difficult problems;
- automate difficult problems;
- communicate ideas, methods, and objectives;
- understand fundamental structures for computer science.

# Course logistics

A quick summary now; details on the course webpage.

# Instructors

## Emina Torlak

Section B: MWF 9:30–10:20 AM

Office hours: Th 12:30-1:30 PM

## Sami Davies

Section A: MWF 1:30–2:20 PM

Office hours: W 2:30-3:30 PM

# TAs

Austin Chan
Daniel Jones
Daniel Thomas Fuchs
Logan Gnanapragasam
Harrison Bay
Kevin Pham
Karishma Ramesh Mandyam
Leana Chen
Louis Maliyam
Mrigank Arora
Oscar Sprumont
Pemi Nguyen
Frank Qin

# Communication

**Zoom**
For lectures, sections, and office hours. Lectures will be recorded.

**Canvas**
For grades, important announcements, and links to Zoom meetings.

**Gradescope**
For submitting your work. You will receive an email invitation.

**Piazza**
To discuss the content of the course. Opt out of Piazza Careers.

**Email**
For other matters, send email to cse311-staff@cs, which will reach both the instructors and the TAs.

# Textbook, work, and grading

**Optional textbook**

Rosen, *Discrete Mathematics and Its Applications*, 6th Edition, McGraw-Hill.

**Homework Assignments**

Due at 11:59:59 PM on the due date.

Collaborate but write up individually.

List your collaborators.

Do not leave with any part of the solution in writing or photographs.

Wait at least 30 minutes before writing your own solution.

**Grading**

Weekly homework assignments: 88%

Final homework assignment: 12%

# About grades

Grades were very important up until now …

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Please relax and **focus on learning**!

- Try to avoid asking "will I lose points if…".
- Most such questions are not worthwhile, either for you or for us.
- If you are really worried, then show more work.

# Propositional logic

A language of reasoning

# What is logic and why do we need it?

Logic is a language, like English or Java, with its own

- *words* and rules for combining words into *sentences* (**syntax**), and
- a way to assign *meaning* to words and sentences (**semantics**).

So why learn another language when we know English and Java?

# Why not use English?

Turn right here.

We saw her duck.

Visiting relatives can be fun.

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Is the visit fun or the relatives who are visiting?

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Does “duck” mean the animal or crouch down?

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Is the visit fun or the relatives who are visiting?

Natural language can be imprecise.

# Why not use Java?

The following method determines …

```
public static boolean mystery(int x) {
    for(int r = 2; r < x; r++) {
        for(int q = 2; q < x; q++) {
            if (r*q == x) {
                return false;
            }
        }
    }
    return (x > 1);
}
```

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… if its input is a prime number.

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… if its input is a prime number.

Programs can be verbose and take a while to understand.

# Logic is both precise and concise!

We need a language of reasoning to

- state sentences more precisely,
- state sentences more concisely, and
- understand sentences more quickly.

# Propositions: the basic building blocks of logic

A *proposition* is a statement that is either true or false.

**All cats are mammals.**
This is a true proposition.

**All mammals are cats.**
This is a false proposition.

# Are these propositions?

$2 + 2 = 5$

$x + 2 = 5$

Who are you?

Pay attention.

Every positive even integer can be written as the sum of two primes.

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Not a proposition because it doesn't have a unique truth value.

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Not a proposition because it's a question rather than a statement.

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**Who are you?**

Not a proposition because it's a question rather than a statement.

**Pay attention.**

Not a proposition because it's a command rather than a statement.

**Every positive even integer can be written as the sum of two primes.**

This is a proposition. Nobody knows its truth value, but it's unique!

# Abstracting atomic propositions with variables

*Propositional variables* represent *atomic propositions* ("words").

By convention, we use lower-case letters for these variables: $p, q, r, \ldots$

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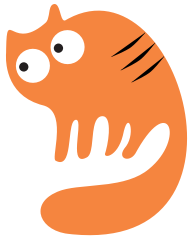
| Variable | Proposition |
| --- | --- |
| $p$ | “Garfield has black stripes.” |
| $q$ | “Garfield is an orange cat.” |
| $r$ | “Garfield likes lasagna.” |



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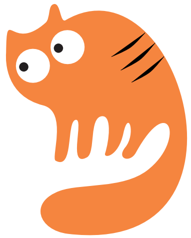

The *truth value* of a propositional variable is either

- T for true, or
- F for false.

# Making compound propositions with logical connectives

We combine atomic propositions into *compound propositions* (“sentences”) using *logical connectives*.

Here is a compound proposition about Garfield:

> *Garfield has black stripes* ***if*** *he is an orange cat* ***and*** *likes lasagna,* ***and*** *he is an orange cat* ***or*** *does* ***not*** *like lasagna.*

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> *Garfield has black stripes **if** he is an orange cat **and** likes lasagna, **and** he is an orange cat **or** does **not** like lasagna.*

Let’s see how to express it in logic using our atomic propositions:

$p$ = “Garfield has black stripes.”
$q$ = “Garfield is an orange cat.”
$r$ = “Garfield likes lasagna.”

# From English to logic

*Garfield has black stripes **if** he is an orange cat **and** likes lasagna, **and** he is an orange cat **or** does **not** like lasagna.*

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*($p$ **if** ($q$ **and** $r$)) **and** ($q$ **or** (**not** $r$))*

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↓ Step 1: abstract

*($p$* ***if*** *($q$* ***and*** *$r$))* ***and*** *($q$* ***or*** *(**not** $r$))*

↓ Step 2: replace English connectives with logical connectives

# Logical connectives

| Connective | Write as | Read as | True when |
|---|---|---|---|
| Negation | $\neg p$ | "not $p$" | $p$ is false |
| Conjunction | $p \wedge q$ | "$p$ and $q$" | both $p$ and $q$ are true |
| Disjunction | $p \vee q$ | "$p$ or $q$" | at least one of $p, q$ is true |
| Exclusive Or | $p \oplus q$ | "either $p$ or $q$" | exactly one of $p, q$ is true |
| Implication | $p \rightarrow q$ | "if $p$ then $q$" | $p$ is false, or both $p, q$ are true |
| Biconditional | $p \leftrightarrow q$ | "$p$ if and only if $q$" | $p, q$ have the same truth value |

*($p$ **if** ($q$ **and** $r$)) **and** ($q$ **or** (**not** $r$))*

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*(p* ***if*** *(q* ***and*** *r))* ***and*** *(q* ***or*** *(****not*** *r))*

$$((q \wedge r) \rightarrow p) \wedge (q \vee (\neg r))$$

# Understanding logical connectives with truth tables

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| $p$ | $\neg p$ |
|---|---|
| F | |
| T | |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| F | F | |
| F | T | |
| T | F | |
| T | T | |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| F | F | |
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| F | T | |
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|---|---|
| F | T |
| T | F |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | T |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| F | F | |
| F | T | |
| T | F | |
| T | T | |

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| F | F | |
| F | T | |
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|---|---|
| F | T |
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|---|---|---|
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | T |

| $p$ | $q$ | $p \vee q$ |
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| F | F | F |
| F | T | T |
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| $p$ | $\neg p$ |
|---|---|
| F | T |
| T | F |

| $p$ | $q$ | $p \wedge q$ |
|---|---|---|
| F | F | F |
| F | T | F |
| T | F | F |
| T | T | T |

| $p$ | $q$ | $p \vee q$ |
|---|---|---|
| F | F | F |
| F | T | T |
| T | F | T |
| T | T | T |

| $p$ | $q$ | $p \oplus q$ |
|---|---|---|
| F | F | F |
| F | T | T |
| T | F | T |
| T | T | F |

# Understanding implication with a truth table

| Connective | Write as | Read as | True when |
|---|---|---|---|
| Implication | $p \to q$ | "if $p$ then $q$" | $p$ is false, or both $p$, $q$ are true |

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | |
| F | T | |
| T | F | |
| T | T | |

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| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

# Understanding implication as promises

It's useful to think of implications as promises. That is "Did I lie?"

> *If it's raining, then I have my umbrella.*

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

| | It's raining | It's not raining |
|---|---|---|
| I have my umbrella | | |
| I don't have my umbrella | | |

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It's useful to think of implications as promises. That is "Did I lie?"

*If it's raining, then I have my umbrella.*

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

| | It's raining | It's not raining |
|---|---|---|
| I have my umbrella | Truth | Truth |
| I don't have my umbrella | Lie | Truth |

The only lie is when:

- It's raining AND
- I don't have my umbrella

# Understanding implication: it's not causal!

Are these true?

**$2 + 2 = 4 \to$ earth is a planet**

**$2 + 2 = 5 \to$ 26 is prime**

| $p$ | $q$ | $p \to q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

# Understanding implication: it's not causal!

Are these true?

**$2 + 2 = 4 \rightarrow$ earth is a planet**

The fact that the atomic propositions "2 + 2 = 4" and "earth is a planet" are unrelated doesn't matter! Both are true, so the implication is true as well.

**$2 + 2 = 5 \rightarrow$ 26 is prime**

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
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**$2 + 2 = 4 \rightarrow$ earth is a planet**

The fact that the atomic propositions "2 + 2 = 4" and "earth is a planet" are unrelated doesn't matter! Both are true, so the implication is true as well.

**$2 + 2 = 5 \rightarrow$ 26 is prime**

Again, the atomic propositions may or may not be related. Because "2 + 2 = 5" is false, the implication is true. Whether 26 is prime or not is irrelevant.

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

# Understanding implication forward and backward

> *1. I have collected all 151 Pokemon if I am a Pokemon master.*
> *2. I have collected all 151 Pokemon only if I am a Pokemon master.*

These sentences are implications in opposite directions:

# Understanding implication forward and backward

> *1. I have collected all 151 Pokemon if I am a Pokemon master.*
> *2. I have collected all 151 Pokemon only if I am a Pokemon master.*

These sentences are implications in opposite directions:

1. Pokemon masters have all 151 Pokemon.
2. People who have 151 Pokemon are Pokemon masters.

So, the implications are:

1. **If** I am a Pokemon master, **then** I have collected all 151 Pokemon.
2. **If** I have collected all 151 Pokemon, **then** I am a Pokemon master.

# Understanding implication some more

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| F | F | T |
| F | T | T |
| T | F | F |
| T | T | T |

- $p$ implies $q$
- whenever $p$ is true $q$ must be true
- if $p$ then $q$
- $q$ if $p$
- $p$ is sufficient for $q$
- $p$ only if $q$
- $q$ is necessary for $p$

# Understanding biconditional (bi-implication)

| Connective | Write as | Read as | True when |
|---|---|---|---|
| Biconditional | $p \leftrightarrow q$ | "$p$ if and only if $q$" | $p, q$ have the same truth value |

| $p$ | $q$ | $p \leftrightarrow q$ |
|---|---|---|
| F | F | |
| F | T | |
| T | F | |
| T | T | |

- $p$ iff $q$
- $p$ is equivalent to $q$
- $p$ implies $q$ and $q$ implies $p$
- $p$ is necessary and sufficient for $q$

# Understanding biconditional (bi-implication)

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| --- | --- | --- | --- |
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| $p$ | $q$ | $p \leftrightarrow q$ |
| --- | --- | --- |
| F | F | T |
| F | T | F |
| T | F | F |
| T | T | T |

- $p$ iff $q$
- $p$ is equivalent to $q$
- $p$ implies $q$ and $q$ implies $p$
- $p$ is necessary and sufficient for $q$

# Now back to understanding our Garfield sentence …

*Garfield has black stripes* ***if*** *he is an orange cat* ***and*** *likes lasagna,* ***and*** *he is an orange cat* ***or*** *does* ***not*** *like lasagna.*

$p$ = "Garfield has black stripes."
$q$ = "Garfield is an orange cat."
$r$ = "Garfield likes lasagna."

↓ Step 1: abstract

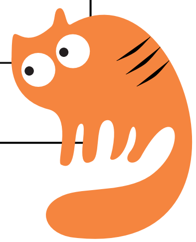
*($p$* ***if*** *($q$* ***and*** *$r$))* ***and*** *($q$* ***or*** *(**not** $r$))*

↓ Step 2: replace English connectives with logical connectives

$((q \wedge r) \to p) \wedge (q \vee (\neg r))$

# Understanding Garfield with a truth table

| $p$ | $q$ | $r$ | $\neg r$ | $(q \vee (\neg r))$ | $(q \wedge r)$ | $((q \wedge r) \rightarrow p)$ | $((q \wedge r) \rightarrow p) \wedge (q \vee (\neg r))$ |
|---|---|---|---|---|---|---|---|
| F | F | F | T | T | F | T | T |
| F | F | T | F | F | F | T | F |
| F | T | F | T | T | F | T | T |
| F | T | T | F | T | T | F | F |
| T | F | F | T | T | F | T | T |
| T | F | T | F | F | F | T | F |
| T | T | F | T | T | F | T | T |
| T | T | T | F | T | T | T | T |



*Garfield has black stripes* ***if*** *he is an orange cat* ***and*** *likes lasagna,* ***and*** *he is an orange cat* ***or*** *does* ***not*** *like lasagna.*

$p$ = "Garfield has black stripes."
$q$ = "Garfield is an orange cat."
$r$ = "Garfield likes lasagna."

# Summary

**Welcome to CSE 311!**

All logistics are on the course webpage.

**Propositional logic lets us be concise and precise.**

Atomic propositions are “words” in propositional logic.

Compound propositions are “sentences” made with logical connectives.

Implication is tricky: when in doubt, write the truth table!