1. **Cantelli’s Rabbits**

Xavier Cantelli owns some rabbits. The number of rabbits he has in any given year is described by the function $f$:

$$
\begin{align*}
  f(0) &= 0 \\
  f(1) &= 1 \\
  f(n) &= 2f(n-1) - f(n-2) \text{ for } n \geq 2
\end{align*}
$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year $n$. That is, construct a formula for $f(n)$ and prove its correctness.

2. **Walk the Dawgs**

Suppose a dog walker takes care of $n \geq 12$ dogs. The dog walker is not a strong person, and will walk dogs in groups of 4 or 5 at a time (every dog gets walked exactly once). Prove the dog walker can always split the $n$ dogs into groups of 4 or 5.

3. **Reversing a Binary Tree**

Consider the following definition of a (binary) Tree.

**Basis Step**\text{ Nil is a Tree.}

**Recursive Step** If $L$ is a Tree, $R$ is a Tree, and $x$ is an integer, then $\text{Tree}(x, L, R)$ is a Tree.

The sum function returns the sum of all elements in a Tree.

$$
\begin{align*}
  \text{sum(\text{Nil})} &= 0 \\
  \text{sum(\text{Tree}(x, L, R))} &= x + \text{sum}(L) + \text{sum}(R)
\end{align*}
$$

The following recursively defined function produces the mirror image of a Tree.

$$
\begin{align*}
  \text{reverse(\text{Nil})} &= \text{Nil} \\
  \text{reverse(\text{Tree}(x, L, R))} &= \text{Tree}(x, \text{reverse}(R), \text{reverse}(L))
\end{align*}
$$

Show that, for all Trees $T$ that

$$
\text{sum}(T) = \text{sum(\text{reverse}(T))}
$$

4. **Bernoulli’s Inequality**

Show that for any integer $n \geq 0$ and real number $x \geq -1$ that $(1 + x)^n \geq 1 + nx$.

5. **Regular Expressions**

(a) Write a regular expression that matches base 10 numbers (e.g., there should be no leading zeroes).

(b) Write a regular expression that matches all base-3 numbers that are divisible by 3.
(c) Write a regular expression that matches all binary strings that contain the substring “111”, but not the substring “000”.