Sets Reference Sheet

Common Sets

- \( \mathbb{N} = \{0, 1, 2, \ldots \} \) is the set of Natural Numbers.
- \( \mathbb{Z} = \{ \ldots, -2, -1, 0, 1, 2, \ldots \} \) is the set of Integers.
- \( \mathbb{Q} = \left\{ \frac{p}{q} : p, q \in \mathbb{Z} \land q \neq 0 \right\} \) is the set of Rational Numbers.
- \( \mathbb{R} \) is the set of Real Numbers.

Containment, Equality, and Subsets

Let \( A, B \) be sets. Then:

- \( x \in A \) ("\( x \) is an element of \( A \)) means that \( x \) is an element of \( A \).
- \( x \notin A \) ("\( x \) is not an element of \( A \)) means that \( x \) is not an element of \( A \).
- \( A \subseteq B \) ("\( A \) is a subset of \( B \)) means that all the elements of \( A \) are also in \( B \).
- \( A \nsubseteq B \) ("\( A \) is not a subset of \( B \)) means that some element of \( A \) is not also in \( B \).
- \( A \supseteq B \) ("\( A \) is a superset of \( B \)) means that all the elements of \( B \) are also in \( A \).
- \( A = B \equiv (A \subseteq B) \land (B \subseteq A) \equiv \forall x \ (x \in A \leftrightarrow x \in B) \)

Set Operations

Let \( A, B \) be sets. Then:

- \( A \cup B \) is the union of \( A \) and \( B \). \( A \cup B = \{ x : x \in A \lor x \in B \} \).
- \( A \cap B \) is the intersection of \( A \) and \( B \). \( A \cap B = \{ x : x \in A \land x \in B \} \).
- \( A \setminus B \) is the difference of \( A \) and \( B \). \( A \setminus B = \{ x : x \in A \land x \notin B \} \).
- \( A \oplus B \) is the symmetric difference of \( A \) and \( B \). \( A \oplus B = \{ x : x \in A \oplus x \in B \} \).
- \( \overline{A} \) is the complement of \( A \). If we restrict ourselves to a “universal set”, \( \mathcal{U} \), (a set of all possible things we're discussing), then \( \overline{A} = \{ x \in \mathcal{U} : x \notin A \} \).

Set Constructions

Let \( A, B, C, D \) be sets. Then:

- \( S = \{ x : P(x) \} \) is set builder notation which means \( S \) is the set that contains all objects \( x \) with property \( P \) (and no other elements).
- \( A \times B \) is the cartesian product of \( A \) and \( B \). \( A \times B = \{ (a, b) : a \in A, \ b \in B \} \).
- \( [n] \) ("brackets \( n \)) is the set of integers from 1 to \( n \). \( [n] = \{ x \in \mathbb{Z} : 1 \leq x \leq n \} \).
- \( \mathcal{P}(A) \) is the power set of \( A \). \( \mathcal{P}(A) = \{ S : S \subseteq A \} \). \( \mathcal{P}(A) \) is the set of all subsets of \( A \).