# Reading 01: Translation Gotchas

In class we gave guidelines for translating between English and predicate logic. In this reading, we justify two of those guidelines by showing how not following the guidelines can go wrong.

## 1. "For Any"

It might seem at first that "for any" makes for a good translation for a universal quantifier (it sounds a lot like "for all" and is usually used in the same way). But there are instances where "for any" can be used to mean an existential quantifier instead of a universal quantifier.

Examples are easier to come by with questions – imagine the following two conversations:

Credit card salesman: You can use credit cards to buy food, household goods, as well as services.
Your internal Monologue: Amazing! So versatile. How far does this go...
You: Can I use a credit card for any purchase?

Cashier: We do not accept credit cards for orders less than $10 or for orders with more than one item.
Your Internal Monologue: But everything here is so expensive...is there anything left?
You: Can I use a credit card for any purchase?

In the first example, we would translate "for any" as a universal quantifier, in the second we would use an existential quantifier. Without the context of the conversation "Can I use a credit card for any purchase?" becomes an ambiguous sentence.

This example should not be fully convincing, though; we don't often need to translate **questions** into logic.

Here is an ambiguous statement, translatable into predicate logic.

I did so well on the midterm, I only need a 20% on the final – I can get an A if I know the solution for any problem on the final.

I didn't do well on the midterm, but that's ok, the final is worth most of the grade – I can get an A if I know the solution for any problem on the final.

In the first sentence, an existential quantifier is intended (at least one correct answer will guarantee a 20%). In the second sentence, a universal quantifier is meant (we need to get a really good grade).

These examples are uncommon, but they can happen, and it is not always easy to take a step back and ensure that someone else will not misinterpret what you have written. For that reason we strongly recommend avoiding "for any" but as long as the sentence remains unambiguous, we will give credit to those translations. While it can be a bad practice, you will certainly see "for any" used in mathematical language. For example, as of this writing, "for any" is used to mean a universal quantifier three times in the Wikipedia article for universal quantification.

## 2. Quantifier Order

We told you that when you are translating sentences with quantifiers back-and-forth from English to logic, you must keep quantifiers in the same order.

Why? Here's a sentence: "Every student had their question answered by some TA."

In common, everyday English this sentence can be interpreted in two ways.

**Interpretation 1:** Wow, the TAs work as an amazing team! Everyone chipped in and by the end of the office hour, every student had their question answered by some TA.

**Interpretation 2:** Every student had their questions answered by some TA – that one TA is a super hero! They must have stayed up all night answering questions...I wonder if I can switch into their section.

The first interpretation is $\forall s \exists t(\texttt{Student}(s) \to [\texttt{TA}(t) \wedge \texttt{AnsweredQuestion}(s,t)])$
The second interpretation is $\exists t \forall s(\texttt{Student}(s) \to [\texttt{TA}(t) \wedge \texttt{AnsweredQuestion}(s,t)])$

In common everyday English, speakers could mean either. In **mathematical** English – the English we use in this class – we need to write sentences like this one **a lot**. Often enough that if we have to pause and look at context (or worse guess) which interpretation was meant we would go crazy.

Instead, we have a convention: we always write the words "for all/each/every" and "there is/exists" in the same order as $\forall$, $\exists$ appear in the logical expression.

So if you wrote that sentence about TAs as part of a proof, it will always be interpreted to mean $\forall s \exists t(\texttt{Student}(s) \to [\texttt{TA}(t) \wedge \texttt{AnsweredQuestion}(s,t)])$. "Every" came first in the sentence, so $\forall$ comes first in the logical syntax.

This convention is followed universally in math and computer science – everyone reading your writing will assume you meant them to translate in order – so make sure that is what you mean.