CSE 311 : Practice Final

This exam is a (slight) modification of a real final given in a prior quarter of CSE311.

The original exam was given in a 110 minute slot.

We strongly recommend you take this exam as though it were closed book – even though your exam will be open book.

Instructions

- Students had 110 minutes to complete the exam.
- The exam was closed resource (except for the logical equivalences, boolean algebra, and inference rules reference sheets). You exam will be open resource.
- The problems are of varying difficulty.
- If you get stuck on a problem, move on and come back to it later.
1. Regularly Irregular [15 points]

Let $\Sigma = \{0, 1\}$. Prove that the language $L = \{x \in \Sigma^* : \#_0(x) < \#_1(x)\}$ is irregular.
2. Recurrences, Recurrences [15 points]

Define

\[ T(n) = \begin{cases} 
  n & \text{if } n = 0, 1 \\
  4T\left(\left\lfloor \frac{n}{2} \right\rfloor \right) + n & \text{otherwise}
\end{cases} \]

Prove that \( T(n) \leq n^3 \) for \( n \geq 3 \).
3. All The Machines! [15 points]

Let \( \Sigma = \{0, 1, 2\} \).
Consider \( L = \{w \in \Sigma^* : \text{Every 1 in the string has at least one 0 before and after it}\} \).

(a) Give a regular expression that represents \( A \).

(b) Give a DFA that recognizes \( A \).

(c) Give a CFG that generates \( A \).
4. Structural CFGs [15 points]

Consider the following CFG: \( S \rightarrow \varepsilon \mid SS \mid S1 \mid S01 \). Another way of writing the recursive definition of this set, \( Q \), is as follows:

- \( \varepsilon \in Q \)
- If \( S \in Q \), then \( S1 \in Q \) and \( S01 \in Q \)
- If \( S, T \in Q \), then \( ST \in Q \).

Prove, by structural induction that if \( w \in Q \), then \( w \) has at least as many 1’s as 0’s.
5. Tralse! [15 points]

For each of the following answer True or False and give a short explanation of your answer.

(a) Any subset of a regular language is also regular.

(b) The set of programs that loop forever on at least one input is decidable.

(c) If $\mathbb{R} \subseteq A$ for some set $A$, then $A$ is uncountable.

(d) If the domain of discourse is people, the logical statement

$$\exists x \ (P(x) \rightarrow \forall y \ (x \neq y \rightarrow \neg P(y)))$$

can be correctly translated as “There exists a unique person who has property $P$”.

(e) $\exists x \ (\forall y \ P(x, y)) \rightarrow \forall y \ (\exists x \ P(x, y))$ is true regardless of what predicate $P$ is.
6. Relationships! [15 points]

The following is the graph of a binary relation $R$.

(a) Draw the transitive-reflexive closure of $R$. [5 points]

(b) Let $S = \{(X, Y) : X, Y \in \mathcal{P}(\mathbb{N}) \land X \subseteq Y\}$.

Recall that $R$ is antisymmetric iff $((a, b) \in R \land a \neq b) \rightarrow (b, a) \notin R$.

Prove that $S$ is antisymmetric. [10 points]
7. Construction Paper! [15 points]

Convert the following NFA into a DFA using the algorithm from lecture.
8. Modern DFAs [15 points]

Let $\Sigma = \{0, 1, 2\}$. Construct a DFA that recognizes exactly strings with a 0 in all positions $i$ where $i \% 3 = 0$. 