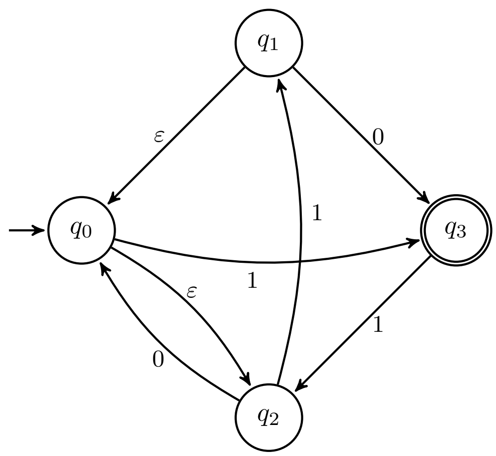
CSE 311 : Extra Final Practice

1. NFA Construction

Draw the state diagram of an NFA M that recognizes the language $a^*b(b \cup ab)^*a$ over $\Sigma = \{a, b\}$. Try to avoid unnecessary states.

2. Powerset Construction

Build a DFA equivalent to the following NFA using the powerset construction. You only need to show states that are reachable from the start state of your DFA (but do not simplify further.



3. DFA, Regexp, CFG

For each of the following langauges, construct a DFA, Regular Expression, and CFG for it.

(a) $A = \{w \in \{0, 1\}^* :$ the number of 0's minus the number of 1's in w is divisible by 3$\}$.

(b) $B = \{w \in \{a, b\}^*$: every a has two b's immediately to its right$\}$.

4. Context-Free & Irregular

Consider the language $C = \{a^n ba^m ba^{m+n} : n, m \geq 1\}$.

(a) Show that C is context-free.

(b) Show that C is not regular.

5. Recursive Definitions & Strong Induction

In the land of Garbanzo, the unit of currency is the bean. They only have two coins, one worth 2 beans and the other worth 5 beans.

(a) Give a recursive definition of the set of positive integers S such that $x \in s$ iff one can make up an amount worth x beans using at most one 5-bean coin and any number of 2-bean coins.

(b) Prove by strong induction that if $n \geq 4$, then $n \in S$.

functions

Define $g(n)$ as follows:

$$g(n+1) = \begin{cases} 0 & \text{if } n = 0 \\ \max\limits_{1 \leq k \leq n} g(k) + g(n+1-k) + 1 & \text{otherwise} \end{cases}$$

Prove by induction that $g(n) = n - 1$ for all $n \geq 1$.

6. Relation Closures

Let R be the relation $\{(1,2),(3,4),(1,3),(2,1)\}$ defined on the set $\{1,2,3,4,5\}$.

(a) Draw the graph of R.

(b) Draw the graph of the R^2.

(c) Draw the graph of the reflexive-transitive closure of R.

7. Relations Proofs

Suppose R_1 and R_2 are reflexive relations on a set A. Is the relation $R_1 \cup R_2$ necessarily a reflexive relation? Justify your answer.

8. True or False

For each of the following answer True or False and give a short (1-2 sentence) explanation of your answer.

(a) The set $\{(\texttt{CODE}(R), x) : R \text{ halts when given } x\}$ is decidable.

(b) The set $\{\texttt{CODE}(Q) : Q \text{ reads input}\}$ is decidable.