CSE 311 : Midterm Exam

Instructions

- This is a take home exam. It is due 72 hours after it is released, i.e. at 11:59 PM on Sunday November 15th, Seattle time (GMT-8).

- Remember to follow the collaboration policy:
  - You are permitted to work with at most 3 other students in the class; you may discuss the exam only with those students and the course staff.
  - As always, you must write up your work independently.

- This exam is open-book/open-note. You may use any resource the staff has provided, as well as external resources.

- But you may not search for the particular problems in the exam.

- Submit your responses to gradescope. You may handwrite or typeset solutions (as long as they are legible). We do not recommend trying to write on this document; we have not left enough room.

- If you have a question, please check the pinned post on Ed where we've posted common clarifications. If you still have a question, please ask a **private** question on ed.

- Unless otherwise noted, we expect English proofs.

Advice

- Remember to properly format English proofs (e.g. introduce all of your variables).

- If you write a symbolic proof, follow the style guidelines on the webpage.

- We give some partial credit for the beginning and end of a proof. Even if you don't know how the middle goes, you can write the start of the proof and put the "target" and conclusion at the bottom.

- Take a deep breath. Gumball says you can do it.

Question	Max points
Translation Tricks	23
Warm-Ups	16
FUNctional Sets	25
Numbers	15
Induction	20
Survey	1
Total	**100**

1.   Translation Tricks [23 points]

For the rest of this page, let your domain of discourse be "books and people". You may use the following predicates for the rest of this page:

- HasRead(p, b) is true iff person p has read book b.
- Enjoyed(p, b) is true iff person p enjoyed book b.
- Author(p, b) is true iff person p is an author of book b.
- SciFi(b) is true iff the book b is a science fiction book.
- KnowsDeMorgans(x) is true iff x is a book where DeMorgan's Laws is stated or x is a person who knows De-Morgan's Laws.
- short(x) is true iff x is a book that is less than 100 pages, or x is a person who is $5'6''$ or shorter.
- book(b) is true iff b is a book.
- person(p) is true iff p is a person.
- $=, \neq$ (with their usual meanings of "equal" and "not equal").

You may use the following two constants: Robbie, Hank Green

1.1.   Reading is Fun! [16 points]

Translate each of these statements into symbolic logic (use predicates and quantifiers). In part (b), you should have one overall expression that represents the whole assertion.

(a) Robbie only enjoys science fiction books.

(b) Both of the following are true:
- Robbie has read a book authored by Hank Green.
- Hank Green has read a book authored by Hank Green.

(c) Hank Green is an author of exactly two books.

(d) Every short book is enjoyed by someone who knows DeMorgan's Laws.

1.2.   Short Books and Short People [7 points]

(a) Translate $\forall x \exists y([\text{person}(x) \land \text{short}(x)] \to [\text{short}(y) \land \text{enjoyed}(x, y)])$ into English. Take advantage of domain restriction. [2 points]

(b) Symbolically negate the sentence above. Simplify until negations apply only to individual predicates. Show your work (but you do not need to name the rules you are using). [3 points]

(c) Write the English version of the negation of the original sentence. Your English translation should take advantage of domain restriction (even if your symbols do not). [2 points]

2. Warm-Ups [16 points]

Recall that a number x is rational if there are integers p and q such that $q \neq 0$ and $x = p/q$.
Equivalently, in symbols: $\texttt{rational}(x) = \exists p \exists q (x = p/q \wedge \texttt{Integer}(p) \wedge \texttt{Integer}(q) \wedge q \neq 0)$.
Call x "irrational" if it is not rational.
For the rest of this page, let your domain of discourse be real numbers. You may use the following predicates $\texttt{rational}(x)$ (as defined above), $\texttt{irrational}(x)$ to mean x is irrational, and $\texttt{Integer}(x)$ to mean "x is an integer."

Consider the statement "If the product of two numbers is irrational, then at least one of the two numbers is irrational."

(a) Write this statement with quantifiers and predicate notation. [3 points]

(b) Prove the statement. You may give an English proof **or** a (symbolic) inference proof.
Give only one proof not both. If you give the symbolic proof, you may use "algebra" as a justification and any rules on the inference rules and logical connectives reference sheets. [9 points]

(c) Give the converse of the original statement in symbols and English.[1] [4 points]

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[1]That is, take the converse of the implication in the statement, leaving the quantifiers unchanged

3. FUNctional Sets [25 points]

For any function f and any set S, define output$(f, S) = \{y : \exists x (x \in S \land f(x) = y)\}$

For example, output$(x^2, \{1, 2, 3\}) = \{y : \exists x (x \in \{1, 2, 3\} \land f(x) = y)\} = \{1, 4, 9\}$,
and output$(x^2, \{-1, 1\}) = \{1\}$.

(a) Show that output$(f, A \cap B) \subseteq$ output$(f, A) \cap$ output(f, B). [10 points]

(b) These two sets are not equal in general. Convince yourself of that fact by considering $A = \{x : x \in \mathbb{Z} \land x < 3\}$, $B = \{x \in \mathbb{Z} \land x > -3\}$ and $f(x) = x^2$. You do not have to write anything for this part. [0 points]

(c) Call a function "non-repetitive" if $\forall x \forall y (f(x) = f(y) \rightarrow x = y)$. Show that x^2 is **not** "non-repetitive" if your domain of discourse is all integers. [3 points]

(d) Show that if a f is non-repetitive, then output$(f, A) \cap$ output$(f, B) \subseteq$ output$(f, A \cap B)$. [12 points]

4. We Went a Little Overboard with the twos... [15 points]

Let a, b be arbitrary integers, and n be an arbitrary positive integer. The following statements look a lot like mod identities, but only one is actually true. Read them carefully.

For the false statement (part a), give an English proof showing it is false.
For the true statement (part b), give an English proof showing it is true.

(a) if $a \equiv b \pmod{n}$ then $a^2 \equiv b^2 \pmod{n^2}$. [3 points]

(b) If $a \equiv b \pmod{n}$ then $2a \equiv 2b \pmod{2n}$. [12 points]

5. It's a Logical Deduction to Use Induction [20 points]

You have n pairs of people to put into a line. In each pair, one person wears a purple hat and the other wears a gold hat. As you put the people in a line, you need to ensure all of the following hold.

- The members of every pair are adjacent.

- The person at the front of the line wears a purple hat.

- The person at the back of the line wears a purple hat.

Call a line "properly ordered" if it meets all of those requirements.
Show that for all $n \geq 2$, in every properly ordered line there are two people wearing gold hats next to each other.

You must do this proof by induction. You may not use proof by contradiction (either for the whole proof or to argue the base case or inductive step).

6. Help us help future you [1 points]

This is the first time 311 has given takehome exams, for our sake (when writing the final) and for the sake of future instructors, it would help to know these pieces of information:

(a) How many hours did you work on this total (estimate to the nearest half-hour)?

(b) How much of that time was just making a readable submission (typing or cleanly writing – estimate to the nearest half-hour)?

(c) How many people did you work with?

(d) On homeworks this quarter, how many people have you usually worked with?

(e) How did you feel about the exam?

6.1. Drawing

Remember T'Vin? The Vulcan child you adopted on HW1? How do you think he's doing? Tell us or draw a picture (any nonempty piece of paper will get credit for this question; looking at these help the TAs stay happy while grading :))