CSE 311: Foundations of Computing I

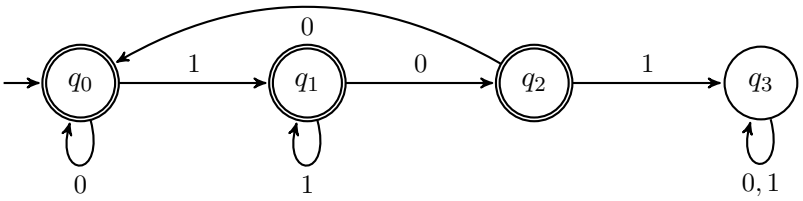
Section 9: DFAs, NFAs, and Minimization Solutions

1. DFAs

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

(a) Strings that do not contain the substring 101.

Solution:



q_3: string that contain 101.

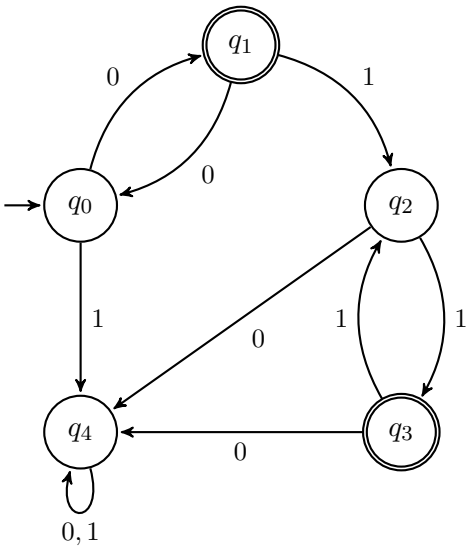
q_2: strings that don't contain 101 and end in 10.

q_1: strings that don't contain 101 and end in 1.

q_0: ε, 0, strings that don't contain 101 and end in 00.

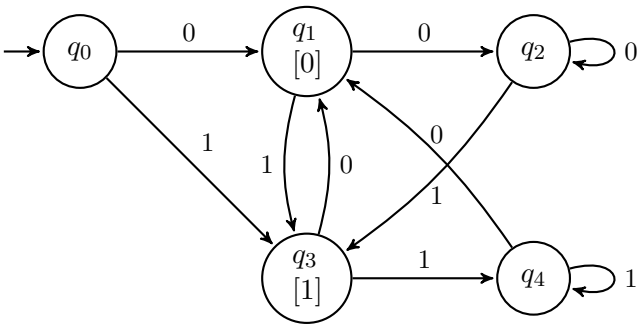
(b) Strings that contain an even number of 1s and odd number of 0's and do not contain the substring 10.

Solution:



2. FSMs with Output

Describe the output of this machine, which operates on binary strings.



Solution:

The states of the machine receiving the following strings:

q_0: Start state only.

q_1: Receives strings ending with 0, where the previous digit does not exist or is a 1.

q_2: Receives strings ending with at least two 0s

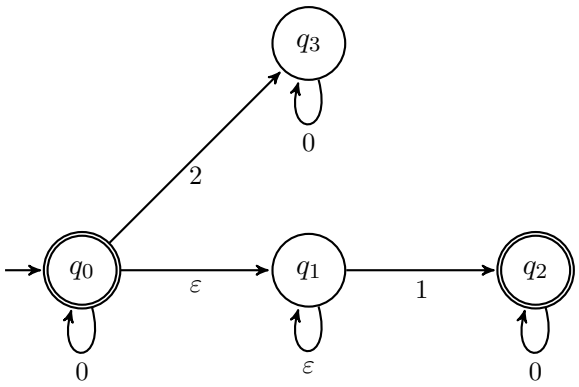
q_3: Receives strings ending with 1, where the previous digit does not exist or is a 0.

q_4: Receives strings ending with at least two 1s.

This outputs a 0 when a first 0 is reached and then none as it sees more 0s directly afterward, and likewise for 1s. In other words, this compresses strings 0s into a single 0 and strings of 1s into a single 1.

3. NFAs

(a) What language does the following NFA accept?



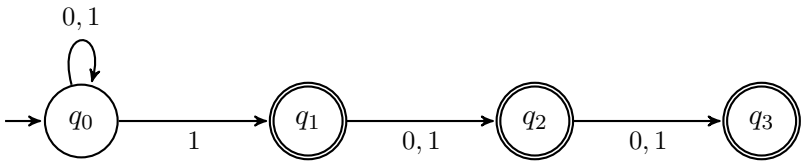
Solution:

All strings of only 0's and 1's not containing more than one 1.

(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

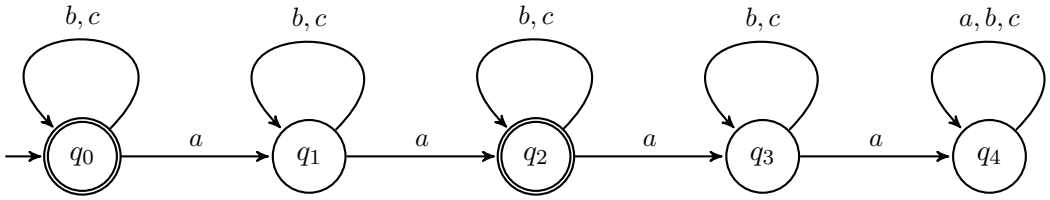
Solution:

The following is one such NFA:



4. DFA Minimization

Minimize the following DFA. For each step of the algorithm write down the groups (of states), which group was split in the step the reason for splitting that group:



Solution:

Step 1: q_0, q_2 are final states and the rest are not final. So, we start with the initial partition with the following groups: group 1 is $\{q_0, q_2\}$ and group 2 is $\{q_1, q_3, q_4\}$.

Step 2: q_1 is sending a to group 1 while q_3, q_4 are sending a to group 2. So, we divide group 2. We get the following groups: group 1 is $\{q_0, q_2\}$, group 3 is $\{q_1\}$ and group 4 is $\{q_3, q_4\}$.

Step 3: q_0 is sending a to group 3 and q_2 is sending a to group 4. So, we divide group 1. We will have the following groups: group 3 is $\{q_1\}$, group 4 is $\{q_3, q_4\}$, group 5 is $\{q_0\}$ and group 6 is $\{q_2\}$.

The minimized DFA is the following: