

CSE 311: Foundations of Computing I

Section 8: CFGs, Relations, and DFAs

1. CFGs

Construct CFGs for the following languages:

- (a) All binary strings that end in 00.
- (b) All binary strings that contain at least three 1's.
- (c) All binary strings with an equal number of 1's and 0's.

2. Relations

- (a) Draw the transitive-reflexive closure of $\{(1, 2), (2, 3), (3, 4)\}$.
- (b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.
- (c) Consider the relation $R = \{(x, y) : x = y + 1\}$ on \mathbb{N} . Is R reflexive? Transitive? Symmetric? Anti-symmetric?
- (d) Consider the relation $S = \{(x, y) : x^2 = y^2\}$ on \mathbb{R} . Prove that S is reflexive, transitive, and symmetric.

3. DFAs

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1, 2, 3\}$.

- (a) All binary strings.
- (b) All strings that contain at least one 3 but no 4.
- (c) All strings whose digits sum to an even number.
- (d) All strings whose digits sum to an odd number.

4. Powers of Relations

Let A be a set and R a relation on A . Use induction to prove that R^n is exactly the pairs of elements from A that are connected by a path of length n in the graph $G = (A, R)$.