CSE 311: Foundations of Computing I

Section 3: Predicate Logic and Inference

1. Quantifier Switch

Consider the following pairs of sentences. For each pair, determine if one implies the other (or if they are equivalent).

(a) $\forall x\, \forall y\, P(x, y)$ $\quad$ $\forall y\, \forall x\, P(x, y)$

(b) $\exists x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \exists x\, P(x, y)$

(c) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\forall y\, \exists x\, P(x, y)$

(d) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists x\, \forall y\, P(x, y)$

(e) $\forall x\, \exists y\, P(x, y)$ $\quad$ $\exists y\, \forall x\, P(x, y)$

2. Formal Proof I

Show that $\neg p \to s$ follows from $p \vee q$, $q \to r$ and $r \to s$.

3. Formal Spoofs

For each of of the following proofs, determine why the proof is incorrect. Then, consider whether the conclusion of the proof is true or not. If it is true, state how the proof could be fixed. If it is false, give a counterexample.

(a) Show that $p \to (q \vee r)$ follows from $p \to q$ and r.

1.	$p \to q$	[Given]
2.	r	[Given]
3.	$p \to (q \vee r)$	[$\vee$ Intro: 1, 2]

(b) Show that q follows from $\neg p \vee q$ and p.

1.	$\neg p \vee q$	[Given]
2.	p	[Given]
3.	q	[$\vee$ Elim: 1, 2]

(c) Show that q follows from $q \vee p$ and $\neg p$.

1.	$\neg p$	[Given]
2.	$q \vee p$	[Given]
3.	$q \vee \mathsf{F}$	[Substitute $p = \mathsf{F}$ since $\neg p$ holds: 1, 2]
4.	q	[Identity: 3]

(d) Show that $\exists z\, \forall x\, P(x, z)$ follows from $\forall x\, \exists y\, P(x, y)$.

1.	$\forall x\, \exists y\, P(x, y)$	[Given]
2.	$\forall x\, P(x, c)$	[$\exists$ Elim: 1, c special]
3.	$\exists z\, \forall x\, P(x, z)$	[$\exists$ Intro: 2]

(e) Show that $\exists z\,(P(z) \wedge Q(z))$ follows from $\forall x\,P(x)$ and $\exists y\,Q(y)$.

1.	$\forall x\,P(x)$	[Given]
2.	$\exists y\,Q(y)$	[Given]
3.	Let z be arbitrary	
4.	$P(z)$	[$\forall$ Elim: 1]
5.	$Q(z)$	[$\exists$ Elim: 2, let z be the object that satisfies $Q(z)$]
6.	$P(z) \wedge Q(z)$	[$\wedge$ Intro: 4, 5]
7.	$\exists z\,(P(z) \wedge Q(z))$	[$\exists$ Intro: 6]

4. Formal Proof II

Show that $\neg p$ follows from $\neg(\neg r \vee t)$, $\neg q \vee \neg s$ and $(p \rightarrow q) \wedge (r \rightarrow s)$.