CSE 311: Foundations of Computing I

Homework 3 (due Wednesday, April 24th at 11:00 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. However, you may use results from lecture, the theorems handout, and previous homeworks without proof.*

Warning: Beginning with Homework 3, graders will deduct points for submissions that are illegible. (Until now, graders have given warnings on those submissions deemed illegible but not deducted points.)

1. I'm Going To Eat You Little Fishy (18 points)

For each of the following English statements, (i) translate it into predicate logic, (ii) write the negation of that statement in predicate logic with the negation symbols pushed as far in as possible so that any negation symbols are directly in front of a predicate, and then (iii) translate the result of (ii) back to (natural) English.

For the logic, let your domain of discourse be cats and food. You should use only the predicates Loves(x, y) and Likes(x, y) which say that cat x loves or likes (respectively) food y; the predicates Cat(x) and Food(x), which say whether x is a cat or food (respectively); and the predicates $x = y$ and $x \neq y$, which say whether x and y are the same object. (Use the constant "Garfield" to refer to Garfield in your logical statements.)

(a) [6 Points] If a cat loves fish and lasagna, then it's Garfield.

(b) [6 Points] There is some food that Garfield likes but no other cat likes.

(c) [6 Points] Every cat likes some food that Garfield does not like.

2. Brimming Over With Wrongability (16 points)

Theorem: Show that s follows from $(\neg q \vee r) \to \neg p$, $\neg p \to \neg r$ and $r \vee (q \wedge s)$.

"Spoof":

1.	$(\neg q \vee r) \to \neg p$	Given
2.	$(\neg q \vee \neg\neg r) \to \neg p$	Double Negation: 1
3.	$\neg(q \wedge \neg r) \to \neg p$	DeMorgan's Law: 2
4.	$p \to (q \wedge \neg r)$	Contrapositive: 3
5.	$p \to \neg r$	$\wedge$ Elim: 4
6.	$\neg p \to \neg r$	Given
7.	$\neg r$	Proof by cases: 5, 6
8.	$r \vee (q \wedge s)$	Given
9.	$q \wedge s$	$\vee$ Elim: 8, 7
10.	s	$\wedge$ Elim: 9

(a) [6 Points] What is the most significant error in this proof? Give the line and briefly explain why it's wrong.

(b) [10 Points] Show that the theorem is true. You do not need to repeat any valid arguments made in the original spoof.

3. Pigs In Space (24 points)

Let p be the proposition "pigs can fly," c be the proposition "cows can whistle," and s be the proposition "the sun rises in the west." (You **should not** consider whether these propositions are true when solving the problem.)

(a) [14 Points] Give a formal proof that $(p \wedge c) \leftrightarrow (p \wedge s)$ follows from $p \rightarrow (c \leftrightarrow s)$.

(b) [6 Points] Use a truth table to show that $(p \wedge c) \leftrightarrow (p \wedge s) \equiv p \rightarrow (c \leftrightarrow s)$.

(c) [2 Points] What else would we need to do, beyond what the work of part (a), in order to establish the equivalence of part (b) using a formal proof rather than a truth table?

(d) [2 Points] Which approach, the one used in part (b) or the one considered in part (c), is preferable in this problem? Why?

4. Q Who (22 points)

Let $Q(x)$ be a predicate defined in this domain.

(a) [10 Points] Give a formal proof that $(\forall y\, Q(y)) \rightarrow \exists x\, (Q(x) \rightarrow \forall y\, Q(y))$.

(b) [10 Points] Give a formal proof that $\neg(\forall y\, Q(y)) \rightarrow \exists x\, (Q(x) \rightarrow \forall y\, Q(y))$.

(c) [2 Points] Why can we conclude from parts (a) and (b) that $\exists x\, (Q(x) \rightarrow \forall y\, Q(y))$ is a tautology?

5. Prime Directive (20 points)

(a) [4 Points] Let n be any positive integer. Briefly explain (no formal proofs) why $n > 1 \equiv \neg(n = 1)$.

(b) [16 Points] Recall that a positive integer p is prime iff there do not exist a positive integers n and m, both greater than 1, such that $p = nm$. (I.e., Prime(p) means $\neg\exists n\, \exists m\, (n > 1 \wedge m > 1 \wedge p = nm)$.)

Give a formal proof of the following: for any prime p, any positive integers n and m satisfying $p = nm$ must satisfy either $n = 1$ or $m = 1$. Feel free to use the equivalence from part (a) as the rule "part (a)".

6. Extra Credit: Space Pirates (0 points)

Five pirates, called Ann, Brenda, Carla, Danielle and Emily, found a treasure of 100 gold coins. On their ship, they decide to split the coins using the following scheme:

- The first pirate in alphabetical order becomes the chief pirate.
- The chief proposes how to share the coins, and all other pirates (excluding the chief) vote for or against it.
- If 50% or more of the pirates vote for it, then the coins will be shared that way.
- Otherwise, the chief will be thrown overboard, and the process is repeated with the pirates that remain.

Thus, in the first round Ann is the chief: if her proposal is rejected, she is thrown overboard and Brenda becomes the chief, etc; if Ann, Brenda, Carla, and Danielle are thrown overboard, then Emily becomes the chief and keeps the entire treasure.

The pirates' first priority is to stay alive: they will act in such a way as to avoid death. If they can stay alive, they want to get as many coins as possible. Finally, they are a blood-thirsty bunch, if a pirate would get the same number of coins if she voted for or against a proposal, she will vote against so that the pirate who proposed the plan will be thrown overboard.

Assuming that all 5 pirates are intelligent (and aware that all the other pirates are just as aware, intelligent, and bloodthirsty), what will happen? Your solution should indicate which pirates die, and how many coins each of the remaining pirates receives.