CSE 311: Foundations of Computing I

Homework 1 (due Wednesday, April 10 at 11:00 PM)

Directions: *Write up carefully argued solutions to the following problems. Your solution should be clear enough that it should explain to someone who does not already understand the answer why it works. However, you may use results from lecture, the theorems handout, and previous homeworks without proof.*

1. I Can't Blab Such Blibber Blubber! (22 points)

Translate these English statements into logical language, making the atomic propositions as small as possible.

(a) [8 Points] Define a set of atomic propositions. Then, use those atomic propositions to turn each of these sentences about (fictional) graduation requirements into logic.

i) You need at least one of the courses "421 algorithms", "416 machine learning", "484 security".

ii) Taking "484 security" requires "421 algorithms".

iii) You cannot take both "421 algorithms" and "416 machine learning".

(b) [8 Points] Define a set of *at most four* atomic propositions. Then, use those atomic propositions to turn each of these sentences about installation requirements into logic.

i) Packet A is not installed only if both B and D are installed.

ii) Packet B is installed unless D is installed or C is not.

iii) Packet B requires A and C.

(c) [6 Points] Translate the following Seattle weather prediction into logic: If it is cold, there will be either rain or fog but not both, and if it is not cold, there won't be fog and either it rains or it does not.

2. Thing One and Thing Two (20 points)

Use truth assignments (i.e., an assignment of a truth value to each variable) to demonstrate that the two propositions in each part are not logically equivalent. Explain why your assignment demonstrates this.

(a) [4 Points] $p \vee q$ vs. $\neg(p \wedge q)$

(b) [4 Points] $\neg(p \oplus q)$ vs. $\neg p \oplus \neg q$

(c) [4 Points] $p \rightarrow (q \rightarrow r)$ vs. $(p \rightarrow q) \rightarrow r$

(d) [4 Points] $(p \rightarrow q) \leftrightarrow r$ vs. $p \leftrightarrow (q \rightarrow r)$

(e) [4 Points] $p \rightarrow ((q \rightarrow r) \rightarrow (p \rightarrow s))$ vs. $p \rightarrow ((q \rightarrow r) \rightarrow (q \rightarrow s))$.

3. Aunt Annie's Alligator (20 points)

Define the three-input "A" gate by the rules $A(r, s, F) = r$ and $A(r, s, T) = s$.

Show how to implement the following gates using only A's. You are allowed to use the constants T and F and the inputs p and q. You are allowed to use inputs multiple times.

You must *justify your answers* if they are not obvious (to the grader).

(a) [4 Points] $\neg p$, using only **one** A gate

(b) [4 Points] $p \vee q$, using only **one** A gate

(c) [4 Points] $p \wedge q$, using only **one** A gate

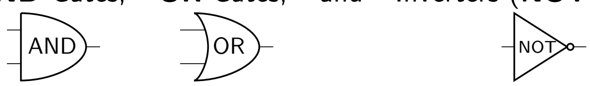
(d) [4 Points] $p \to q$, using only **one** A gates

(e) [4 Points] $\neg p \wedge \neg q$, using at most **two** A gates

4. Would You, Could You, With a Gate? (10 points)

Using only. . .

AND Gates, **OR** Gates, and Inverters (**NOT** Gates),



draw the diagram of a circuit with **three inputs** that computes the function $B(p, q, r)$, defined as follows. If p is false or q is false, then $B(p, q, r) = r$. Otherwise, $B(p, q, r) = \neg r$.

5. One, Two Three... How Many Fingers Do I See? (10 points)

Find a compound proposition involving the propositional variables p, q, and r that is true precisely when a majority of p, q, and r are true. Explain why your answer works.

6. The Curious Case of The Lying TAs (10 points)

A new UW CSE student wandered around the Paul Allen building on their first day in the major. They found (as many do) that there is a secret room in its basements. On the door of this secret room is a sign that says:

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All ye who enter, beware!  Every inhabitant of this room is either a TA who always
 lies or a student who always tells the truth!
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(a) [5 Points] The CSE student walked into the room, and two inhabitants walked up to the student. One of them said "at least one of us is a TA." Determine (with justification) all the possibilities for each of the two inhabitants.

(b) [5 Points] Three inhabitants walk up to the CSE student and surround the UW CSE student. One of them says "every TA in this circle has a TA to his immediate right." Determine (with justification) all the possibilities for each of the three inhabitants.

7. EXTRA CREDIT: XNORing (0 points)

For two bits a and b, we define $\texttt{XNOR}(a, b) = \neg(a \oplus b)$. You are given two memory registers, each with the same number of bits. You have an operation, $\texttt{XNOR}(R_1, R_2)$, which takes two registers, R_1 and R_2, performs bitwise `XNOR` between them, and stores the result in R_1. Show how you can swap the contents of the two registers using a sequence of `XNOR`s without temporary memory registers.