CSE 311: Foundations of Computing I

Section 9: Relations and DFAs

1. Relations

(a) Draw the transitive-reflexive closure of $\{(1,2),(2,3),(3,4)\}$.

(b) Suppose that R is reflexive. Prove that $R \subseteq R^2$.

(c) Consider the relation $R = \{(x,y) \ : \ x = y+1\}$ on $\mathbb{N}$.
Is R reflexive? Transitive? Symmetric? Anti-symmetric?

(d) Consider the relation $S = \{(x,y) \ : \ x^2 = y^2\}$ on $\mathbb{R}$. Prove that S is reflexive, transitive, and symmetric.

2. DFAs

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0,1,2,3\}$.

(a) All binary strings.

(b) All strings that contain at least one 3 but no 2.

(c) All strings whose digits sum to an even number.

(d) All strings whose digits sum to an odd number.

3. DFAs II

Construct DFAs to recognize each of the following languages. Let $\Sigma = \{0, 1\}$.

(a) Strings that do not contain the substring 101.

(b) Strings that contain an even number of 1s and odd number of 0's and do not contain the substring 10.

4. Powers of Relations

Let A be a set and R a relation on A. Use induction to prove that R^n is exactly the pairs of elements from A that are connected by a path of length n in the graph $G = (A, R)$.