CSE 311: Foundations of Computing I

Section 7: Strong Induction and Recursive Sets

1. Binary Representations

Prove that every natural number can be written as a sum of *distinct* powers of two. (I.e., that it has a unique binary representation.)

2. Cantelli's Rabbits

Xavier Cantelli owns some rabbits. The number of rabbits he has in a given year is described by the function f:

$$f(0) = 0$$
$$f(1) = 1$$
$$f(n) = 2f(n-1) - f(n-2) \qquad \text{for } n \geq 2$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year n.

3. Recursively Defined Sets of Strings

For each of the following, write a recursive definition of the sets satisfying the following properties. Briefly justify that your solution is correct.

(a) Binary strings of even length.

(b) Binary strings not containing 10.

(c) Binary strings not containing 10 as a substring and having at least as many 1s as 0s.

(d) Binary strings containing at most two 0s and at most two 1s.