CSE 311: Foundations of Computing I

Section 6: Induction

1. Extended Euclidean Algorithm

(a) Find the multiplicative inverse y of 7 mod 33. That is, find y such that $7y \equiv 1 \pmod{33}$. You should use the extended Euclidean Algorithm. Your answer should be in the range $0 \leq y < 33$.

(b) Now, solve $7z \equiv 2 \pmod{33}$.

2. Induction with Sums: Equality

For any $n \in \mathbb{N}$, define S_n to be the sum of the squares of the first n positive integers, or

$$S_n = \sum_{i=1}^{n} i^2.$$

For all $n \in \mathbb{N}$, prove that $S_n = \frac{1}{6}n(n+1)(2n+1)$.

3. A Strict Inequality

Prove that $6n + 6 < 2^n$ for all $n \geq 6$.

4. Another Inequality

Prove that, for all integers $n \geq 1$, if you have numbers $a_1, \cdots, a_n$ and $b_1, \cdots, b_n$, with $\forall i \in [n].\ a_i \leq b_i$, then:

$$\sum_{i=1}^{n} a_i \leq \sum_{i=1}^{n} b_i$$