CSE 311: Foundations of Computing I

Section 4: English Proofs and Sets

1. Formal Spoofs

For each of of the following proofs, determine why the proof is incorrect. Then, consider whether the conclusion of the proof is true or not. If it is true, state how the proof could be fixed. If it is false, give a counterexample.

(a) Show that $\exists z\,\forall x\,P(x,z)$ follows from $\forall x\,\exists y\,P(x,y)$.

1.	$\forall x\,\exists y\,P(x,y)$	[Given]
2.	$\forall x\,P(x,c)$	[$\exists$ Elim: 1, c special]
3.	$\exists z\,\forall x\,P(x,z)$	[$\exists$ Intro: 2]

(b) Show that $\exists z\,(P(z) \wedge Q(z))$ follows from $\forall x\,P(x)$ and $\exists y\,Q(y)$.

1.	$\forall x\,P(x)$	[Given]
2.	$\exists y\,Q(y)$	[Given]
3.	Let z be arbitrary	
4.	$P(z)$	[$\forall$ Elim: 1]
5.	$Q(z)$	[$\exists$ Elim: 2, let z be the object that satisfies $Q(z)$]
6.	$P(z) \wedge Q(z)$	[$\wedge$ Intro: 4, 5]
7.	$\exists z\,(P(z) \wedge Q(z))$	[$\exists$ Intro: 6]

2. Game, Set, Match

Let A, B, and C be arbitrary sets. Consider the claim that $A \setminus B \subseteq A \cup C$.

(a) Write a formal proof of the claim.

(b) Translate your formal proof to an English proof.

3. Ghosts and Skeletons

Let A and B be sets and P and Q be predicates. For each of the claims below, write the *skeleton* of an English proof of the claim. It will not be possible to complete the proof with just the information given, but you should be able to see the basic shape of the proof.

(a) $A = B$

(b) No element of A satisfies P.

(c) Any object that satisfies P but not Q is in the set B.

(d) B is not a subset of A.

4. Primality Checking

When checking if a number n is prime by brute force, it is only necessary to check possible factors up to $\sqrt{n}$.

Specifically, we can show the following. Let n, a, and b be positive integers. Here is a proof that, if $n = ab$, then either a or b is at most $\sqrt{n}$.

1.1.	$n = ab$		Assumption
	1.2.1 $\neg(a \le \sqrt{n} \lor b \le \sqrt{n})$	Assumption	
	1.2.2 $(a > \sqrt{n}) \land (b > \sqrt{n})$	De Morgan: 1.2.1	
	1.2.3 $a > \sqrt{n}$	Elim $\land$: 1.2.2	
	1.2.4 $b > \sqrt{n}$	Elim $\land$: 1.2.2	
	1.2.5 $ab > \sqrt{n}\sqrt{n} = n$	Prop of ">": 1.2.3, 1.2.4 Algebra	
	1.2.6 $(ab = n) \land (ab > n)$	Intro $\land$: 1.1, 1.2.5	
	1.2.7 F	Prop of ">", 1.2.6	
1.2.	$\neg(a \le \sqrt{n} \lor b \le \sqrt{n}) \to$ F		Direct Proof
1.3.	$\neg\neg(a \le \sqrt{n} \lor b \le \sqrt{n}) \lor$ F		Law of Implication: 1.2
1.4.	$\neg\neg(a \le \sqrt{n} \lor b \le \sqrt{n})$		Identity: 1.3
1.5.	$a \le \sqrt{n} \lor b \le \sqrt{n}$		Double Negation: 1.4
1.	$(n = ab) \to (a \le \sqrt{n} \lor b \le \sqrt{n})$		Direct Proof

Translate this formal proof to an English proof. (Hint: notice which of the proof strategies is being used in part of this proof. Our proof strategies each have special, often shorter, English translations.)