CSE 311: Foundations of Computing I

Section 2: Equivalences and Boolean Algebra

1. Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a) $\neg p \to (q \to r)$ $\qquad$ $q \to (p \vee r)$

(b) $p \leftrightarrow q$ $\qquad$ $(p \wedge q) \vee (\neg p \wedge \neg q)$

2. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a) $p \to q$ $\qquad$ $q \to p$

(b) $p \to (q \wedge r)$ $\qquad$ $(p \to q) \wedge r$

3. Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

(a) $\neg p \vee (\neg q \vee (p \wedge q))$

(b) $\neg(p \vee (q \wedge p))$

4. Properties of XOR

Like $\wedge$ and $\vee$, the $\oplus$ operator (exclusive or) has many interesting properties. For example, it is easy to verify with a truth table that $\oplus$ is also associative. In this problem, we will prove some additional properties of $\oplus$.

Use equivalence chains to prove each of the facts stated below. For this problem only, you may also use the equivalence

$$p \oplus q \equiv (p \wedge \neg q) \vee (\neg p \wedge q)$$

which you may cite as "Definition of $\oplus$". This equivalence allows you to translate $\oplus$ into an expression involving only $\wedge$, $\vee$, and $\neg$, so that the standard equivalences can then be applied.

(a) $p \oplus q \equiv q \oplus p$ (Commutativity)

(b) $p \oplus p \equiv \mathsf{F}$ and $p \oplus \neg p \equiv \mathsf{T}$

(c) $p \oplus \mathsf{F} \equiv p$ and $p \oplus \mathsf{T} \equiv \neg p$

(d) $(\neg p) \oplus q \equiv \neg(p \oplus q) \equiv p \oplus (\neg q)$. I.e., negating one of the inputs negates the overall expression.