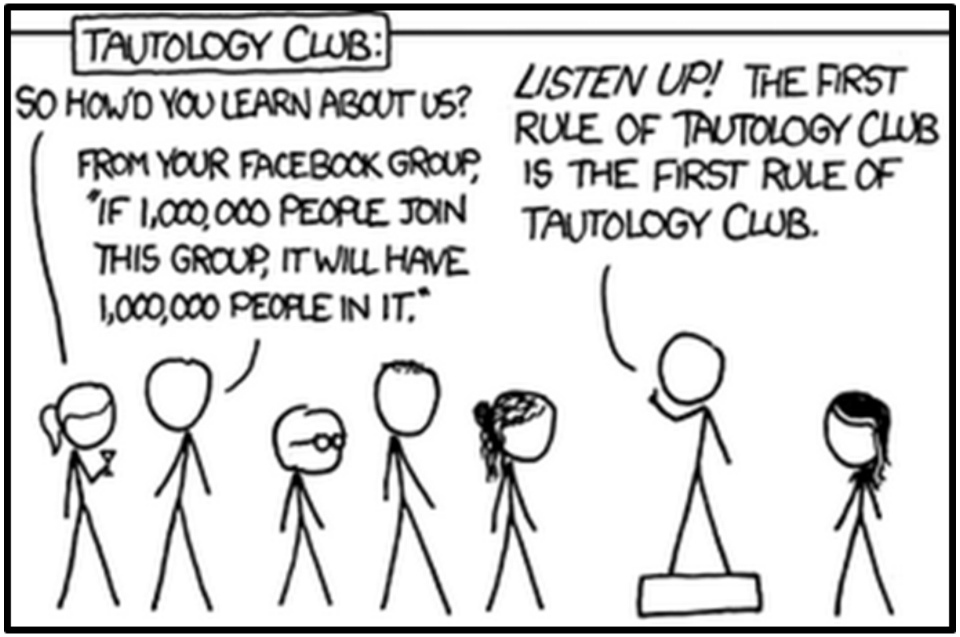
CSE 311: Foundations of Computing

Lecture 4: Boolean Algebra, Circuits, Canonical Forms


TAUTOLOGY CLUB:
SO HOW'D YOU LEARN ABOUT US?
FROM YOUR FACEBOOK GROUP, "IF 1,000,000 PEOPLE JOIN THIS GROUP, IT WILL HAVE 1,000,000 PEOPLE IN IT."
LISTEN UP! THE FIRST RULE OF TAUTOLOGY CLUB IS THE FIRST RULE OF TAUTOLOGY CLUB.

Last Time: Proofs of Equivalence

To show A is equivalent to B

- Apply a series of logical equivalences to sub-expressions to convert A to B

To show A is a tautology

- Apply a series of logical equivalences to sub-expressions to convert A to **T**

Prove this is a Tautology: Option 1

Associative
- $(p \vee q) \vee r \equiv p \vee (q \vee r)$
- $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

Distributive
- $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
- $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Absorption
- $p \vee (p \wedge q) \equiv p$
- $p \wedge (p \vee q) \equiv p$

Negation
- $p \vee \neg p \equiv \mathrm{T}$
- $p \wedge \neg p \equiv \mathrm{F}$

$$(p \wedge q) \rightarrow (q \vee p)$$

Use a series of equivalences like so:

$(p \wedge q) \rightarrow (q \vee p) \equiv \neg(p \wedge q) \vee (q \vee p)$	Law of Implication
$\equiv (\neg p \vee \neg q) \vee (q \vee p)$	DeMorgan
$\equiv \neg p \vee (\neg q \vee (q \vee p))$	Associative
$\equiv \neg p \vee ((\neg q \vee q) \vee p)$	Associative
$\equiv \neg p \vee (p \vee (\neg q \vee q))$	Commutative
$\equiv (\neg p \vee p) \vee (\neg q \vee q)$	Associative
$\equiv (p \vee \neg p) \vee (q \vee \neg q)$	Commutative (twice)
$\equiv \mathrm{T} \vee \mathrm{T}$	Negation (twice)
$\equiv \mathrm{T}$	Domination/Identity

Identity
- $p \wedge \mathrm{T} \equiv p$
- $p \vee \mathrm{F} \equiv p$

Domination
- $p \vee \mathrm{T} \equiv \mathrm{T}$
- $p \wedge \mathrm{F} \equiv \mathrm{F}$

Idempotent
- $p \vee p \equiv p$
- $p \wedge p \equiv p$

Commutative
- $p \vee q \equiv q \vee p$
- $p \wedge q \equiv q \wedge p$

Prove this is a Tautology: Option 2

$$(p \wedge q) \rightarrow (q \vee p)$$

Make a Truth Table and show:

$$(p \wedge q) \rightarrow (q \vee p) \equiv \mathbf{T}$$

$\boldsymbol{p}$	$\boldsymbol{q}$	$\boldsymbol{p \wedge q}$	$\boldsymbol{q \vee p}$	$\boldsymbol{(p \wedge q) \rightarrow (q \vee p)}$
T	**T**	**T**	**T**	**T**
T	**F**	**F**	**T**	**T**
F	**T**	**F**	**T**	**T**
F	**F**	**F**	**F**	**T**

Boolean Logic

Combinational Logic

– output = F(input)

Sequential Logic

– $\text{output}_t = F(\text{output}_{t-1}, \text{input}_t)$

- output dependent on history
- concept of a time step (clock, t)

Boolean Logic

Combinational Logic

- output = F(input)

Sequential Logic

- $\text{output}_t = F(\text{output}_{t-1}, \text{input}_t)$
  - output dependent on history
  - concept of a time step (clock, t)

Boolean Algebra: Another notation for logic consisting of…

- a set of elements B = {0, 1}
- binary operations { + , • } (OR, AND)
- and a unary operation { ’ } (NOT)

Boolean Algebra

- Usual notation used in circuit design

- Boolean algebra
  - a set of elements B containing {0, 1}
  - binary operations { + , • }
  - and a unary operation { ’ }
  - such that the following axioms hold:

For any a, b, c in B:

1. closure:	$a + b$ is in B	$a \bullet b$ is in B
2. commutativity:	$a + b = b + a$	$a \bullet b = b \bullet a$
3. associativity:	$a + (b + c) = (a + b) + c$	$a \bullet (b \bullet c) = (a \bullet b) \bullet c$
4. distributivity:	$a + (b \bullet c) = (a + b) \bullet (a + c)$	$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
5. identity:	$a + 0 = a$	$a \bullet 1 = a$
6. complementarity:	$a + a' = 1$	$a \bullet a' = 0$
7. null:	$a + 1 = 1$	$a \bullet 0 = 0$
8. idempotency:	$a + a = a$	$a \bullet a = a$
9. involution:	$(a')' = a$	

A Combinational Logic Example

Sessions of Class:

We would like to compute the number of lectures or quiz sections remaining *at the start* of a given day of the week.

- **Inputs:** Day of the Week, Lecture/Section flag
- **Output:** Number of sessions left

Examples: Input: (Wednesday, Lecture) Output: 2

Input: (Monday, Section) Output: 1

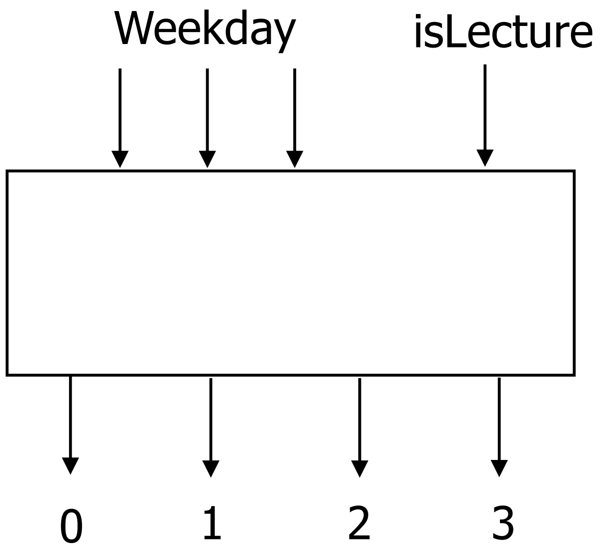
Implementation in Software

```
public int classesLeftInMorning(weekday, isLecture) {
    switch (weekday) {
        case SUNDAY:
        case MONDAY:
            return isLecture ? 3 : 1;
        case TUESDAY:
        case WEDNESDAY:
            return isLecture ? 2 : 1;
        case THURSDAY:
            return isLecture ? 1 : 1;
        case FRIDAY:
            return isLecture ? 1 : 0;
        case SATURDAY:
            return isLecture ? 0 : 0;
  }
}
```

Implementation with Combinational Logic

Encoding:

- How many bits for each input/output?
- Binary number for weekday
- One bit for each possible output



Defining Our Inputs!

Weekday Input:

- Binary number for weekday
- Sunday = 0, Monday = 1, …
- We care about these in binary:

Weekday	Number	Binary
Sunday	0	$(000)_2$
Monday	1	$(001)_2$
Tuesday	2	$(010)_2$
Wednesday	3	$(011)_2$
Thursday	4	$(100)_2$
Friday	5	$(101)_2$
Saturday	6	$(110)_2$

Converting to a Truth Table!

```
case SUNDAY or MONDAY:
   return isLecture ? 3 : 1;
case TUESDAY or WEDNESDAY:
   return isLecture ? 2 : 1;
case THURSDAY:
   return isLecture ? 1 : 1;
case FRIDAY:
   return isLecture ? 1 : 0;
case SATURDAY:
   return isLecture ? 0 : 0;
```

Weekday		isLecture	c_0	c_1	c_2	c_3
SUN	000	0				
SUN	000	1				
MON	001	0				
MON	001	1				
TUE	010	0				
TUE	010	1				
WED	011	0				
WED	011	1				
THU	100	-				
FRI	101	0				
FRI	101	1				
SAT	110	-				
-	111	-				

Converting to a Truth Table!

```
case SUNDAY or MONDAY:
   return isLecture ? 3 : 1;
case TUESDAY or WEDNESDAY:
   return isLecture ? 2 : 1;
case THURSDAY:
   return isLecture ? 1 : 1;
case FRIDAY:
   return isLecture ? 1 : 0;
case SATURDAY:
   return isLecture ? 0 : 0;
```

Weekday		isLecture	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

Truth Table to Logic (Part 1)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

Let's begin by finding an expression for $\mathbf{c_3}$. To do this, we look at the rows where $\mathbf{c_3}$ = **1** (true).

Truth Table to Logic (Part 1)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

DAY == SUN && L == 1

DAY == MON && L == 1

Truth Table to Logic (Part 1)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

$d_2d_1d_0$ == 000 && L == 1

$d_2d_1d_0$ == 001 && L == 1

Substituting DAY for the binary representation.

Truth Table to Logic (Part 1)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

d_2 == 0 && d_1 == 0 && d_0 == 0 && L == 1

d_2 == 0 && d_1 == 0 && d_0 == 1 && L == 1

Splitting up the bits of the day; so, we can write a formula.

Truth Table to Logic (Part 1)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

$d_2' \cdot d_1' \cdot d_0' \cdot L$

$d_2' \cdot d_1' \cdot d_0 \cdot L$

Replacing with Boolean Algebra…

Truth Table to Logic (Part 1)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

$d_2' \cdot d_1' \cdot d_0' \cdot L$

$d_2' \cdot d_1' \cdot d_0 \cdot L$

Either situation causes c_3 to be true. So, we “or” them.

$$c_3 = d_2' \cdot d_1' \cdot d_0' \cdot L + d_2' \cdot d_1' \cdot d_0 \cdot L$$

Truth Table to Logic (Part 2)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

$$c_3 = d_2' \bullet d_1' \bullet d_0' \bullet L + d_2' \bullet d_1' \bullet d_0 \bullet L$$

Now, we do c_2.

Truth Table to Logic (Part 3)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

Now, we do **c_1**:

$$c_3 = d_2' \bullet d_1' \bullet d_0' \bullet L + d_2' \bullet d_1' \bullet d_0 \bullet L$$

$$c_2 = d_2' \bullet d_1 \bullet d_0' \bullet L + d_2' \bullet d_1 \bullet d_0 \bullet L$$

Truth Table to Logic (Part 3)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3	Now, we do c_1:
SUN	000	0	0	1	0	0	$d_2' \cdot d_1' \cdot d_0' \cdot L'$
SUN	000	1	0	0	0	1	
MON	001	0	0	1	0	0	$d_2' \cdot d_1' \cdot d_0 \cdot L'$
MON	001	1	0	0	0	1	
TUE	010	0	0	1	0	0	$d_2' \cdot d_1 \cdot d_0' \cdot L'$
TUE	010	1	0	0	1	0	
WED	011	0	0	1	0	0	$d_2' \cdot d_1 \cdot d_0 \cdot L'$
WED	011	1	0	0	1	0	
THU	100	-	0	1	0	0	???
FRI	101	0	1	0	0	0	
FRI	101	1	0	1	0	0	$d_2 \cdot d_1' \cdot d_0 \cdot L$
SAT	110	-	1	0	0	0	
-	111	-	1	0	0	0	

$$c_3 = d_2' \cdot d_1' \cdot d_0' \cdot L + d_2' \cdot d_1' \cdot d_0 \cdot L$$

$$c_2 = d_2' \cdot d_1 \cdot d_0' \cdot L + d_2' \cdot d_1 \cdot d_0 \cdot L$$

Truth Table to Logic (Part 3)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3	
SUN	000	0	0	1	0	0	$d_2' \cdot d_1' \cdot d_0' \cdot L'$
SUN	000	1	0	0	0	1	
MON	001	0	0	1	0	0	$d_2' \cdot d_1' \cdot d_0 \cdot L'$
MON	001	1	0	0	0	1	
TUE	010	0	0	1	0	0	$d_2' \cdot d_1 \cdot d_0' \cdot L'$
TUE	010	1	0	0	1	0	
WED	011	0	0	1	0	0	$d_2' \cdot d_1 \cdot d_0 \cdot L'$
WED	011	1	0	0	1	0	
THU	100	-	0	1	0	0	$d_2 \cdot d_1' \cdot d_0'$
FRI	101	0	1	0	0	0	
FRI	101	1	0	1	0	0	$d_2 \cdot d_1' \cdot d_0 \cdot L$
SAT	110	-	1	0	0	0	
-	111	-	1	0	0	0	

Now, we do c_1:

No matter what L is, we always say it's 1. So, we don't need L in the expression.

$$c_3 = d_2' \cdot d_1' \cdot d_0' \cdot L + d_2' \cdot d_1' \cdot d_0 \cdot L$$

$$c_2 = d_2' \cdot d_1 \cdot d_0' \cdot L + d_2' \cdot d_1 \cdot d_0 \cdot L$$

Truth Table to Logic (Part 3)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

Now, we do c_1:

$d_2' \cdot d_1' \cdot d_0' \cdot L'$

$d_2' \cdot d_1' \cdot d_0 \cdot L'$

$d_2' \cdot d_1 \cdot d_0' \cdot L'$

$d_2' \cdot d_1 \cdot d_0 \cdot L'$

$d_2 \cdot d_1' \cdot d_0'$

$d_2 \cdot d_1' \cdot d_0 \cdot L$

No matter what L is, we always say it’s 1. So, we don’t need L in the expression.

$$c_3 = d_2' \cdot d_1' \cdot d_0' \cdot L + d_2' \cdot d_1' \cdot d_0 \cdot L$$

$$c_2 = d_2' \cdot d_1 \cdot d_0' \cdot L + d_2' \cdot d_1 \cdot d_0 \cdot L$$

$$c_1 = d_2' \cdot d_1' \cdot d_0' \cdot L' + d_2' \cdot d_1' \cdot d_0 \cdot L' + d_2' \cdot d_1 \cdot d_0' \cdot L' + d_2' \cdot d_1 \cdot d_0 \cdot L' + d_2 \cdot d_1' \cdot d_0' + d_2 \cdot d_1' \cdot d_0 \cdot L$$

Truth Table to Logic (Part 4)

	$d_2d_1d_0$	L	c_0	c_1	c_2	c_3
SUN	000	0	0	1	0	0
SUN	000	1	0	0	0	1
MON	001	0	0	1	0	0
MON	001	1	0	0	0	1
TUE	010	0	0	1	0	0
TUE	010	1	0	0	1	0
WED	011	0	0	1	0	0
WED	011	1	0	0	1	0
THU	100	-	0	1	0	0
FRI	101	0	1	0	0	0
FRI	101	1	0	1	0	0
SAT	110	-	1	0	0	0
-	111	-	1	0	0	0

$$c_1 = d_2' \bullet d_1' \bullet d_0' \bullet L' + d_2' \bullet d_1' \bullet d_0 \bullet L' + d_2' \bullet d_1 \bullet d_0' \bullet L' + d_2' \bullet d_1 \bullet d_0 \bullet L' + d_2 \bullet d_1' \bullet d_0' + d_2 \bullet d_1' \bullet d_0 \bullet L$$

$$c_2 = d_2' \bullet d_1 \bullet d_0' \bullet L + d_2' \bullet d_1 \bullet d_0 \bullet L$$

$$c_3 = d_2' \bullet d_1' \bullet d_0' \bullet L + d_2' \bullet d_1' \bullet d_0 \bullet L$$

Finally, we do c_0:

$d_2 \bullet d_1' \bullet d_0 \bullet L'$

$d_2 \bullet d_1 \bullet d_0'$

$d_2 \bullet d_1 \bullet d_0$

Truth Table to Logic (Part 4)

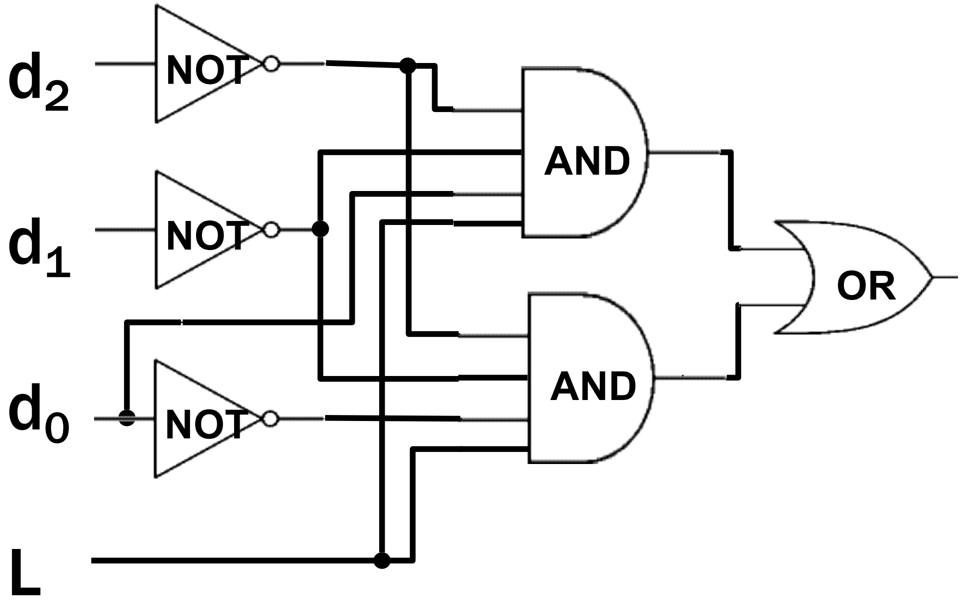
$c_0 = d_2 \bullet d_1' \bullet d_0 \bullet L' + d_2 \bullet d_1 \bullet d_0' + d_2 \bullet d_1 \bullet d_0$

$c_1 = d_2' \bullet d_1' \bullet d_0' \bullet L' + d_2' \bullet d_1' \bullet d_0 \bullet L' + d_2' \bullet d_1 \bullet d_0' \bullet L' + d_2' \bullet d_1 \bullet d_0 \bullet L' + d_2 \bullet d_1' \bullet d_0' + d_2 \bullet d_1' \bullet d_0 \bullet L$

$c_2 = d_2' \bullet d_1 \bullet d_0' \bullet L + d_2' \bullet d_1 \bullet d_0 \bullet L$

$c_3 = d_2' \bullet d_1' \bullet d_0' \bullet L + d_2' \bullet d_1' \bullet d_0 \bullet L$

Here's c_3 as a circuit:



Boolean Algebra

- Usual notation used in circuit design
- Boolean algebra
  - a set of elements B containing {0, 1}
  - binary operations { + , • }
  - and a unary operation { ’ }
  - such that the following axioms hold:

For any a, b, c in B:

1. closure:	$a + b$ is in B	$a \bullet b$ is in B
2. commutativity:	$a + b = b + a$	$a \bullet b = b \bullet a$
3. associativity:	$a + (b + c) = (a + b) + c$	$a \bullet (b \bullet c) = (a \bullet b) \bullet c$
4. distributivity:	$a + (b \bullet c) = (a + b) \bullet (a + c)$	$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
5. identity:	$a + 0 = a$	$a \bullet 1 = a$
6. complementarity:	$a + a' = 1$	$a \bullet a' = 0$
7. null:	$a + 1 = 1$	$a \bullet 0 = 0$
8. idempotency:	$a + a = a$	$a \bullet a = a$
9. involution:	$(a')' = a$	

Simplification using Boolean Algebra

uniting:

10. $a \bullet b + a \bullet b' = a$

10D. $(a + b) \bullet (a + b') = a$

absorption:

11. $a + a \bullet b = a$

11D. $a \bullet (a + b) = a$

12. $(a + b') \bullet b = a \bullet b$

12D. $(a \bullet b') + b = a + b$

factoring:

13. $(a + b) \bullet (a' + c) =$
$a \bullet c + a' \bullet b$

13D. $a \bullet b + a' \bullet c =$
$(a + c) \bullet (a' + b)$

consensus:

14. $(a \bullet b) + (b \bullet c) + (a' \bullet c) =$
$a \bullet b + a' \bullet c$

14D. $(a + b) \bullet (b + c) \bullet (a' + c) =$
$(a + b) \bullet (a' + c)$

de Morgan’s:

15. $(a + b + \ldots)' = a' \bullet b' \bullet \ldots$

15D. $(a \bullet b \bullet \ldots)' = a' + b' + \ldots$

Proving Theorems

2. commutativity:	$a + b = b + a$	$a \bullet b = b \bullet a$
3. associativity:	$a + (b + c) = (a + b) + c$	$a \bullet (b \bullet c) = (a \bullet b) \bullet c$
4. distributivity:	$a + (b \bullet c) = (a + b) \bullet (a + c)$	$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
5. identity:	$a + 0 = a$	$a \bullet 1 = a$
6. complementarity:	$a + a' = 1$	$a \bullet a' = 0$
7. null:	$a + 1 = 1$	$a \bullet 0 = 0$
8. idempotency:	$a + a = a$	$a \bullet a = a$
9. involution:	$(a')' = a$	

Using the laws of Boolean Algebra:

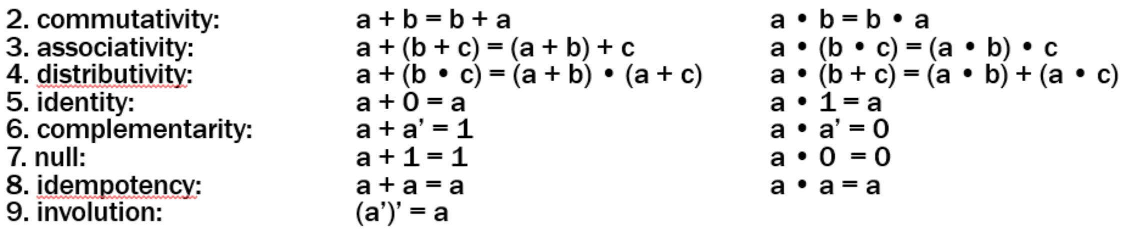
prove the Uniting theorem: $X \bullet Y + X \bullet Y' = X$

$$X \bullet Y + X \bullet Y' =$$

prove the Absorption theorem: $X + X \bullet Y = X$

$$X + X \bullet Y =$$

Proving Theorems



2. commutativity:	$a + b = b + a$	$a \bullet b = b \bullet a$
3. associativity:	$a + (b + c) = (a + b) + c$	$a \bullet (b \bullet c) = (a \bullet b) \bullet c$
4. distributivity:	$a + (b \bullet c) = (a + b) \bullet (a + c)$	$a \bullet (b + c) = (a \bullet b) + (a \bullet c)$
5. identity:	$a + 0 = a$	$a \bullet 1 = a$
6. complementarity:	$a + a' = 1$	$a \bullet a' = 0$
7. null:	$a + 1 = 1$	$a \bullet 0 = 0$
8. idempotency:	$a + a = a$	$a \bullet a = a$
9. involution:	$(a')' = a$	

Using the laws of Boolean Algebra:

prove the Uniting theorem: $X \bullet Y + X \bullet Y' = X$

distributivity	$X \bullet Y + X \bullet Y' = X \bullet (Y + Y')$
complementarity	$= X \bullet 1$
identity	$= X$

prove the Absorption theorem: $X + X \bullet Y = X$

identity	$X + X \bullet Y = X \bullet 1 + X \bullet Y$
distributivity	$= X \bullet (1 + Y)$
commutativity	$= X \bullet (Y + 1)$
null	$= X \bullet 1$
identity	$= X$

Proving Theorems

Using truth table:

For example, de Morgan’s Law:

(X + Y)′ = X′ • Y′

NOR is equivalent to AND with inputs complemented

X	Y	X′	Y′	(X + Y)′	X′ • Y′
0	0	1	1	1	1
0	1	1	0	0	0
1	0	0	1	0	0
1	1	0	0	0	0

(X • Y)′ = X′ + Y′

NAND is equivalent to OR with inputs complemented

X	Y	X′	Y′	(X • Y)′	X′ + Y′
0	0	1	1	1	1
0	1	1	0	1	1
1	0	0	1	1	1
1	1	0	0	0	0

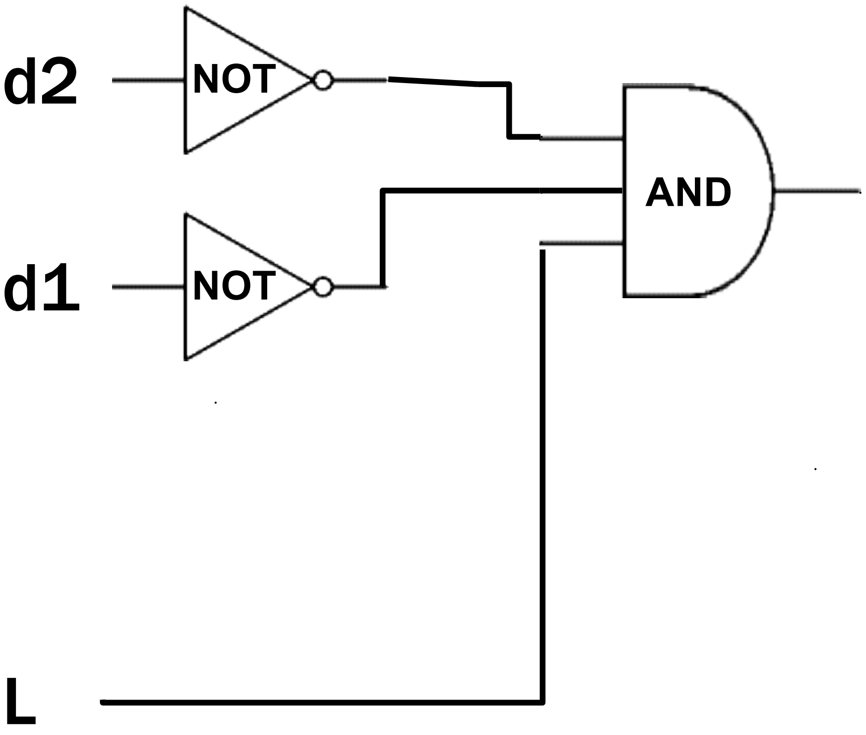
Simplifying using Boolean Algebra

c3 = d2’•d1’•d0’•L + d2’•d1’•d0•L

= d2’•d1’•(d0’ + d0)•L

= d2’•d1’•1•L

= d2’•d1’•L


d2
NOT
d1
NOT
AND
L