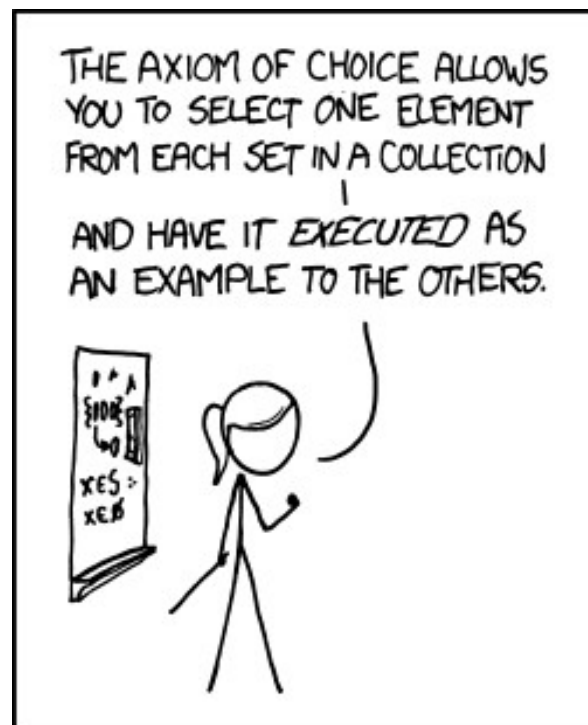


CSE 311: Foundations of Computing

Lecture 8: Predicate Logic Proofs



MY MATH TEACHER WAS A BIG
BELIEVER IN PROOF BY INTIMIDATION.

Last class: Propositional Inference Rules

Two inference rules per binary connective, one to eliminate it and one to introduce it

$$\boxed{\text{Elim } \wedge} \frac{A \wedge B}{\therefore A, B}$$

$$\boxed{\text{Intro } \wedge} \frac{A ; B}{\therefore A \wedge B}$$

$$\boxed{\text{Elim } \vee} \frac{A \vee B ; \neg A}{\therefore B}$$

$$\boxed{\text{Intro } \vee} \frac{A}{\therefore A \vee B, B \vee A}$$

$$\boxed{\text{Modus Ponens}} \frac{A ; A \rightarrow B}{\therefore B}$$

$$\boxed{\text{Direct Proof Rule}} \frac{A \Rightarrow B}{\therefore A \rightarrow B}$$

Not like other rules

Last class: Example

Prove: $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

1.1. $(p \rightarrow q) \wedge (q \rightarrow r)$ Assumption

1.2. $p \rightarrow q$ \wedge Elim: 1.1

1.3. $q \rightarrow r$ \wedge Elim: 1.1

1.4.1. p Assumption

1.4.2. q MP: 1.2, 1.4.1

1.4.3. r MP: 1.3, 1.4.2

1.4. $p \rightarrow r$ Direct Proof Rule

1. $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$ Direct Proof Rule

One General Proof Strategy

- 1. Look at the rules for introducing connectives to see how you would build up the formula you want to prove from pieces of what is given**
- 2. Use the rules for eliminating connectives to break down the given formulas so that you get the pieces you need to do 1.**
- 3. Write the proof beginning with what you figured out for 2 followed by 1.**

Inference Rules for Quantifiers: First look

$$\boxed{\text{Intro } \exists} \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\boxed{\text{Elim } \forall} \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

$$\boxed{\text{Intro } \forall} \frac{\text{“Let } a \text{ be arbitrary”} \dots P(a)}{\therefore \forall x P(x)}$$

$$\boxed{\text{Elim } \exists} \frac{\exists x P(x)}{\therefore P(c) \text{ for some } \textit{special}^{**} c}$$

* in the domain of P

** By special, we mean that c is a name for a value where P(c) is true. We can't use anything else about that value, so c has to be a NEW name!

Predicate Logic Proofs

- **Can use**
 - **Predicate logic inference rules**
whole formulas only
 - **Predicate logic equivalences (De Morgan's)**
even on subformulas
 - **Propositional logic inference rules**
whole formulas only
 - **Propositional logic equivalences**
even on subformulas

My First Predicate Logic Proof

Prove $\forall x P(x) \rightarrow \exists x P(x)$

$$\text{Intro } \exists \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\text{Elim } \forall \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

5. $\forall x P(x) \rightarrow \exists x P(x)$



The main connective is implication
so Direct Proof Rule seems good

My First Predicate Logic Proof

$$\text{Intro } \exists \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\text{Elim } \forall \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

Prove $\forall x P(x) \rightarrow \exists x P(x)$

1.1. $\forall x P(x)$ Assumption

We need an \exists we don't have
so "intro \exists " rule makes sense

1.5. $\exists x P(x)$



1. $\forall x P(x) \rightarrow \exists x P(x)$ Direct Proof Rule

My First Predicate Logic Proof

Prove $\forall x P(x) \rightarrow \exists x P(x)$

Intro \exists $\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$

Elim \forall $\frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$

1.1. $\forall x P(x)$ Assumption

We need an \exists we don't have
so "intro \exists " rule makes sense

1.5. $\exists x P(x)$

Intro \exists :  That requires $P(c)$
for some c .

1. $\forall x P(x) \rightarrow \exists x P(x)$ Direct Proof Rule

My First Predicate Logic Proof

Prove $\forall x P(x) \rightarrow \exists x P(x)$

Intro \exists $\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$

Elim \forall $\frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$

1.1. $\forall x P(x)$


Assumption

1.2. $P(a)$

Elim \forall : 1.1

We could have picked any name
or domain expression here.

1.5. $\exists x P(x)$

Intro \exists : 

That requires $P(c)$
for some c .

1. $\forall x P(x) \rightarrow \exists x P(x)$

Direct Proof Rule

My First Predicate Logic Proof

$$\text{Intro } \exists \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\text{Elim } \forall \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

Prove $\forall x P(x) \rightarrow \exists x P(x)$

No holes. Just need to clean up.

1.1. $\forall x P(x)$ Assumption

1.2 $P(a)$ Elim \forall : 1.1

1.5. $\exists x P(x)$ Intro \exists : 1.2

1. $\forall x P(x) \rightarrow \exists x P(x)$ Direct Proof Rule

My First Predicate Logic Proof

Prove $\forall x P(x) \rightarrow \exists x P(x)$

Intro \exists $\frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$

Elim \forall $\frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$

- | | | |
|------|------------------|-----------------------|
| 1.1. | $\forall x P(x)$ | Assumption |
| 1.2 | $P(a)$ | Elim \forall : 1.1 |
| 1.3. | $\exists x P(x)$ | Intro \exists : 1.2 |

1. $\forall x P(x) \rightarrow \exists x P(x)$ Direct Proof Rule

Working forwards as well as backwards:

In applying “Intro \exists ” rule we didn’t know what expression we might be able to prove $P(c)$ for, so we worked forwards to figure out what might work.

Predicate Logic Proofs with more content

- In propositional logic we could just write down other propositional logic statements as “givens”
- Here, we also want to be able to use domain knowledge so proofs are about something specific

- Example:

Domain of Discourse
Integers

- Given the basic properties of arithmetic on integers, define:

Predicate Definitions
$\text{Even}(x) \equiv \exists y (x = 2 \cdot y)$
$\text{Odd}(x) \equiv \exists y (x = 2 \cdot y + 1)$

A Not so Odd Example

Domain of Discourse

Integers

Predicate Definitions

Even(x) $\equiv \exists y (x = 2 \cdot y)$

Odd(x) $\equiv \exists y (x = 2 \cdot y + 1)$

Prove “There is an even number”

Formally: prove $\exists x \text{ Even}(x)$

A Not so Odd Example

Domain of Discourse

Integers

Predicate Definitions

$\text{Even}(x) \equiv \exists y (x = 2 \cdot y)$

$\text{Odd}(x) \equiv \exists y (x = 2 \cdot y + 1)$

Prove “There is an even number”

Formally: prove $\exists x \text{ Even}(x)$

1. $2 = 2 \cdot 1$ Arithmetic
2. $\exists y (2 = 2 \cdot y)$ Intro \exists : 1
3. $\text{Even}(2)$ Definition of Even: 2
4. $\exists x \text{ Even}(x)$ Intro \exists : 3

A Prime Example

Domain of Discourse

Integers

Predicate Definitions

$\text{Even}(x) \equiv \exists y (x = 2 \cdot y)$

$\text{Odd}(x) \equiv \exists y (x = 2 \cdot y + 1)$

$\text{Prime}(x) \equiv "x > 1 \text{ and } x \neq a \cdot b \text{ for}$
all integers a, b with $1 < a < x"$

Prove "There is an even prime number"

A Prime Example

Domain of Discourse

Integers

Predicate Definitions

$\text{Even}(x) \equiv \exists y (x = 2 \cdot y)$

$\text{Odd}(x) \equiv \exists y (x = 2 \cdot y + 1)$

$\text{Prime}(x) \equiv$ “ $x > 1$ and $x \neq a \cdot b$ for
all integers a, b with $1 < a < x$ ”

Prove “There is an even prime number”

Formally: prove $\exists x (\text{Even}(x) \wedge \text{Prime}(x))$

1. $2 = 2 \cdot 1$

Arithmetic

2. $\text{Prime}(2)^*$

Property of integers

* Later we will further break down “Prime” using quantifiers to prove statements like this

A Prime Example

Domain of Discourse

Integers

Predicate Definitions

$\text{Even}(x) \equiv \exists y (x = 2 \cdot y)$

$\text{Odd}(x) \equiv \exists y (x = 2 \cdot y + 1)$

$\text{Prime}(x) \equiv$ “ $x > 1$ and $x \neq a \cdot b$ for
all integers a, b with $1 < a < x$ ”

Prove “There is an even prime number”

Formally: prove $\exists x (\text{Even}(x) \wedge \text{Prime}(x))$

- | | | |
|----|---|-----------------------|
| 1. | $2 = 2 \cdot 1$ | Arithmetic |
| 2. | $\text{Prime}(2)^*$ | Property of integers |
| 3. | $\exists y (2 = 2 \cdot y)$ | Intro \exists : 1 |
| 4. | $\text{Even}(2)$ | Defn of Even: 3 |
| 5. | $\text{Even}(2) \wedge \text{Prime}(2)$ | Intro \wedge : 2, 4 |
| 6. | $\exists x (\text{Even}(x) \wedge \text{Prime}(x))$ | Intro \exists : 5 |

* Later we will further break down “Prime” using quantifiers to prove statements like this

Inference Rules for Quantifiers: First look

$$\boxed{\text{Intro } \exists} \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\boxed{\text{Elim } \forall} \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

$$\boxed{\text{Intro } \forall} \frac{\text{“Let } a \text{ be arbitrary”} \dots P(a)}{\therefore \forall x P(x)}$$

$$\boxed{\text{Elim } \exists} \frac{\exists x P(x)}{\therefore P(c) \text{ for some } \textit{special}^{**} c}$$

* in the domain of P

** By special, we mean that c is a name for a value where P(c) is true. We can't use anything else about that value, so c has to be a NEW name!

Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

Prove: “The square of every even number is even.”

Formal proof of: $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

3. $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$



Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

Prove: “The square of every even number is even.”

Formal proof of: $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

1. Let **a** be an arbitrary integer

2. $\text{Even}(a) \rightarrow \text{Even}(a^2)$

3. $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$



Intro \forall : 1,2

Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

<p>1. Let a be an arbitrary integer</p> <p>2.1 $Even(a)$ Assumption</p> <p>2.6 $Even(a^2)$</p> <p>2. $Even(a) \rightarrow Even(a^2)$ Direct proof rule</p> <p>3. $\forall x (Even(x) \rightarrow Even(x^2))$ Intro \forall: 1,2</p>	<p>Elim \exists $\exists x P(x)$</p> <hr/> <p>$\therefore P(c)$ for some <i>special</i> c</p>
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Prove: “The square of every even number is even.”

Formal proof of: $\forall x (Even(x) \rightarrow Even(x^2))$

1. Let a be an arbitrary integer

2.1 $Even(a)$ Assumption

2.6 $Even(a^2)$

2. $Even(a) \rightarrow Even(a^2)$

3. $\forall x (Even(x) \rightarrow Even(x^2))$



Direct proof rule

Intro \forall : 1,2

Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

Prove: “The square of every even number is even.”

Formal proof of: $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

1. Let **a** be an arbitrary integer

2.1 $\text{Even}(\mathbf{a})$ Assumption

2.2 $\exists y (\mathbf{a} = 2y)$ Definition of Even

2.5 $\exists y (\mathbf{a}^2 = 2y)$

2.6 $\text{Even}(\mathbf{a}^2)$

2. $\text{Even}(\mathbf{a}) \rightarrow \text{Even}(\mathbf{a}^2)$

3. $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$



Definition of Even

Direct proof rule

Intro \forall : 1,2

Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

Prove: “The square of every even number is even.”

Formal proof of: $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

1. Let **a** be an arbitrary integer

2.1 $\text{Even}(\mathbf{a})$ Assumption

2.2 $\exists y (\mathbf{a} = 2y)$ Definition of Even

2.5 $\exists y (\mathbf{a}^2 = 2y)$ Intro \exists rule:  Need $\mathbf{a}^2 = 2c$ for some **c**

2.6 $\text{Even}(\mathbf{a}^2)$ Definition of Even

2. $\text{Even}(\mathbf{a}) \rightarrow \text{Even}(\mathbf{a}^2)$ Direct proof rule

3. $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$ Intro \forall : 1,2

Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

Prove: “The square of every even number is even.”

Formal proof of: $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

1. Let **a** be an arbitrary integer

2.1 $\text{Even}(\mathbf{a})$

Assumption

2.2 $\exists y (\mathbf{a} = 2y)$

Definition of Even

2.3 $\mathbf{a} = 2\mathbf{b}$

Elim \exists : **b** special depends on **a**

2.5 $\exists y (\mathbf{a}^2 = 2y)$

Intro \exists rule: 

Need $\mathbf{a}^2 = 2\mathbf{c}$
for some **c**

2.6 $\text{Even}(\mathbf{a}^2)$

Definition of Even

2. $\text{Even}(\mathbf{a}) \rightarrow \text{Even}(\mathbf{a}^2)$

Direct proof rule

3. $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

Intro \forall : 1,2

Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

Prove: “The square of every even number is even.”

Formal proof of: $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

1. Let **a** be an arbitrary integer

2.1 $\text{Even}(\mathbf{a})$

Assumption

2.2 $\exists y (\mathbf{a} = 2y)$

Definition of Even

2.3 $\mathbf{a} = 2\mathbf{b}$

Elim \exists : **b** special depends on **a**

2.4 $\mathbf{a}^2 = 4\mathbf{b}^2 = 2(2\mathbf{b}^2)$

Algebra

2.5 $\exists y (\mathbf{a}^2 = 2y)$

Intro \exists rule

Used $\mathbf{a}^2 = 2c$ for $c=2\mathbf{b}^2$

2.6 $\text{Even}(\mathbf{a}^2)$

Definition of Even

2. $\text{Even}(\mathbf{a}) \rightarrow \text{Even}(\mathbf{a}^2)$

Direct proof rule

3. $\forall x (\text{Even}(x) \rightarrow \text{Even}(x^2))$

Intro \forall : 1,2

Why did we need to say that **b** depends on **a**?

There are extra conditions on using these rules:

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

* in the domain of P

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

** c has to be a NEW name.

Over integer domain: $\forall x \exists y (y \geq x)$ is **True** but $\exists y \forall x (y \geq x)$ is **False**

BAD “PROOF”

1. $\forall x \exists y (y \geq x)$ Given
2. Let **a** be an arbitrary integer
3. $\exists y (y \geq \mathbf{a})$ Elim \forall : 1
4. $\mathbf{b} \geq \mathbf{a}$ Elim \exists : **b** special depends on **a**
5. $\forall x (\mathbf{b} \geq x)$ Intro \forall : 2,4
6. $\exists y \forall x (y \geq x)$ Intro \exists : 5

Why did we need to say that **b** depends on **a**?

There are extra conditions on using these rules:

Intro \forall “Let **a** be arbitrary*” ...P(**a**)
 $\therefore \forall x P(x)$

* in the domain of P

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** **c**

** **c** has to be a NEW name.

Over integer domain: $\forall x \exists y (y \geq x)$ is **True** but $\exists y \forall x (y \geq x)$ is **False**

BAD “PROOF”

- | | | |
|----|--------------------------------------|---|
| 1. | $\forall x \exists y (y \geq x)$ | Given |
| 2. | Let a be an arbitrary integer | |
| 3. | $\exists y (y \geq \mathbf{a})$ | Elim \forall : 1 |
| 4. | $\mathbf{b} \geq \mathbf{a}$ | Elim \exists : b special depends on a |
| 5. | $\forall x (\mathbf{b} \geq x)$ | Intro \forall : 2,4 |
| 6. | $\exists y \forall x (y \geq x)$ | Intro \exists : 5 |

Can't get rid of **a** since another name in the same line, **b**, depends on it!

Why did we need to say that **b** depends on **a**?

There are extra conditions on using these rules:

Intro \forall “Let a be arbitrary*” ...P(a)
 $\therefore \forall x P(x)$

* in the domain of P. No other name in P depends on a

Elim \exists $\exists x P(x)$
 $\therefore P(c)$ for some *special*** c

** c is a NEW name.
 List all dependencies for c.

Over integer domain: $\forall x \exists y (y \geq x)$ is **True** but $\exists y \forall x (y \geq x)$ is **False**

BAD “PROOF”

- | | | |
|----|---|---|
| 1. | $\forall x \exists y (y \geq x)$ | Given |
| 2. | Let a be an arbitrary integer | |
| 3. | $\exists y (y \geq \mathbf{a})$ | Elim \forall : 1 |
| 4. | $\mathbf{b} \geq \mathbf{a}$ | Elim \exists : b special depends on a |
| 5. | $\forall x (\mathbf{b} \geq x)$ | Intro \forall: 2,4 |
| 6. | $\exists y \forall x (y \geq x)$ | Intro \exists : 5 |

Can't get rid of **a** since another name in the same line, **b**, depends on it!

Inference Rules for Quantifiers: Full version

$$\boxed{\text{Intro } \exists} \frac{P(c) \text{ for some } c}{\therefore \exists x P(x)}$$

$$\boxed{\text{Elim } \forall} \frac{\forall x P(x)}{\therefore P(a) \text{ for any } a}$$

$$\boxed{\text{Intro } \forall} \frac{\text{“Let } a \text{ be arbitrary”}^* \dots P(a)}{\therefore \forall x P(x)}$$

* in the domain of P. No other name in P depends on a

$$\boxed{\text{Elim } \exists} \frac{\exists x P(x)}{\therefore P(c) \text{ for some } \textit{special}^{**} c}$$

** c is a NEW name.
List all dependencies for c.

English Proofs

- **We often write proofs in English rather than as fully formal proofs**
 - They are more natural to read
- **English proofs follow the structure of the corresponding formal proofs**
 - Formal proof methods help to understand how proofs really work in English...
 - ... and give clues for how to produce them.

An English Proof

Predicate Definitions

$$\text{Even}(x) \equiv \exists y (x = 2 \cdot y)$$

$$\text{Odd}(x) \equiv \exists y (x = 2 \cdot y + 1)$$

Prove “There is an even integer”

Proof:

$$2 = 2 \cdot 1$$



1. $2 = 2 \cdot 1$

Arithmetic

so 2 equals 2 times an integer.



2. $\exists y (2 = 2 \cdot y)$

Intro \exists : 1

Therefore 2 is even.



3. $\text{Even}(2)$

Defn of Even: 2

Therefore, there is an even integer ■



4. $\exists x \text{Even}(x)$

Intro \exists : 3

English Even and Odd

Even(x) $\equiv \exists y (x=2y)$
Odd(x) $\equiv \exists y (x=2y+1)$
Domain: Integers

Prove “The square of every even integer is even.”

Proof: Let **a** be an arbitrary even integer.

1. Let **a** be an arbitrary integer

2.1 **Even(a)** Assumption

Then, by definition, **a = 2b** for some integer **b** (depending on **a**).

2.2 $\exists y (a = 2y)$ Definition

2.3 **a = 2b** **b** special depends on **a**

Squaring both sides, we get **a² = 4b² = 2(2b²)**.

2.4 **a² = 4b² = 2(2b²)** Algebra

Since **2b²** is an integer, by definition, **a²** is even.

2.5 $\exists y (a^2 = 2y)$

2.6 **Even(a²)** Definition

Since **a** was arbitrary, it follows that the square of every even number is even. ■

2. **Even(a) \rightarrow Even(a²)**

3. **$\forall x (Even(x) \rightarrow Even(x^2))$**

Even and Odd

Predicate Definitions

$$\text{Even}(x) \equiv \exists y (x = 2y)$$

$$\text{Odd}(x) \equiv \exists y (x = 2y + 1)$$

Domain of Discourse

Integers

Prove “The square of every odd number is odd.”

Even and Odd

Predicate Definitions

$$\text{Even}(x) \equiv \exists y (x = 2y)$$

$$\text{Odd}(x) \equiv \exists y (x = 2y + 1)$$

Domain of Discourse

Integers

Prove “The square of every odd number is odd.”

Proof: Let b be an arbitrary odd number.

Then, $b = 2c+1$ for some integer c (depending on b).

Therefore, $b^2 = (2c+1)^2 = 4c^2 + 4c + 1 = 2(2c^2 + 2c) + 1$.

Since $2c^2+2c$ is an integer, b^2 is odd. The statement follows since b was arbitrary. ■

Proofs

- **Formal proofs follow simple well-defined rules and should be easy to check**
 - In the same way that code should be easy to execute
- **English proofs correspond to those rules but are designed to be easier for humans to read**
 - Easily checkable in principle