Lecture 1: Propositional Logic
Some Perspective

Computer Science and Engineering

Programming

Theory

Hardware

CSE 14x

CSE 311
About the Course

We will study the theory needed for CSE:

Logic:
How can we describe ideas precisely?

Formal Proofs:
How can we be positive we’re correct?

Number Theory:
How do we keep data secure?

Relations/Relational Algebra:
How do we store information?

Finite State Machines:
How do we design hardware and software?

Turing Machines:
Are there problems computers can’t solve?
About the Course

It’s about perspective!

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives...
- Tools for automating difficult problems
- Fundamental structures for computer science
About the Course

It’s about perspective!

• Tools for reasoning about difficult problems
• Tools for communicating ideas, methods, objectives...
• Tools for automating difficult problems
• Fundamental structures for computer science

This is NOT a programming course!
Instructors

Paul Beame
Section B
MWF 10:30-11:20 in MUE 153
Office Hours:
MF 11:30-12:00 and M 3:00-4:00
CSE 668

Kevin Zatloukal
Section A
MWF 1:30-2:20 in SIG 134
Office Hours:
WF 2:30-3:00 and M 12:00-1:00
CSE 212

Office hours are for students in both sections
TAs and Administrivia

Teaching Assistants:
- Darin Chin
- Daniel Fuchs
- Yuqi Huang
- Joy Ji
- Kaiyu Zheng
- Joshua Fan
- Kush Gupta
- Sean Jaffe
- Cheng Ni

Homework:
- Due WED at 11:59 pm online
- Write up individually
- Extra Credit

Grading (roughly):
- 50% Homework
- 15-20% Midterm
- 30-35% Final Exam

(Optional) Book:
- Rosen: Readings for 6th (used) or 7th (cut down) editions.
- Good for practice with solved problems

Section:
- Thursdays
- starting this week

Office Hours: TBA

All Course Information @ cs.uw.edu/311
CSE 311: Foundations of Computing I
Spring, 2018

Paul Beame
Section A: MWF 10:30-11:20, MUE 153
Office hours: MF 11:30-12:00 and TBA
CSE 668

Kevin Zatloukal
Section B: MWF 1:30-2:20, SIG 134
Office hours: TBA
CSE 312

Email and discussion:
email list: cse311-spl18 [archives]
Please send any e-mail about the course to cse311-staff@cs.

Discussion Board (moderated by TBA)
Use this board to discuss the content of the course. That includes everything except the solutions to current homework problems. Feel free to discuss homeworks and exams from past incarnations of the course, and any confusion over topics discussed in class. It is also acceptable to ask for clarifications about the statement of homework problems, but not about their solutions.

Textbook:
There is no required text for the course. Especially over the first 6-7 weeks of the course, the following textbook can be a useful companion: Rosen, Discrete Mathematics and Its Applications, McGraw-Hill. We will support two versions equally: (1) a special reduced version of the 7th edition which at $60 costs 1/4 of the ridiculous price of the full text, and (2) the 6th edition, which is available used through the bookstore for even less money. It should also be available on short-term loan from the Engineering Library.

Course Calendar

<table>
<thead>
<tr>
<th>#</th>
<th>date</th>
<th>topic</th>
<th>slides</th>
<th>inked (A)</th>
<th>inked (B)</th>
<th>reading (Rosen)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Mon, Mar 26</td>
<td>Propositional Logic</td>
<td>pdf</td>
<td>1.1, 1.2</td>
<td>1.1, 6.5</td>
<td>7th 1.1, 6.5</td>
</tr>
<tr>
<td>2</td>
<td>Wed, Mar 28</td>
<td>Logic/Gates</td>
<td></td>
<td>1.1-1.3</td>
<td>1.1-1.2</td>
<td>6th 1.1-1.2</td>
</tr>
<tr>
<td>3</td>
<td>Fri, Mar 30</td>
<td>More Logic/Circuits</td>
<td></td>
<td>1.2-1.3</td>
<td>1.1-1.3</td>
<td>6th 1.1-1.3</td>
</tr>
<tr>
<td>4</td>
<td>Mon, Apr 2</td>
<td>Boolean Algebra/Circuits</td>
<td></td>
<td>1.2-1.3</td>
<td>1.1-1.3</td>
<td>6th 1.1-1.3</td>
</tr>
<tr>
<td>5</td>
<td>Wed, Apr 4</td>
<td>Canonical Forms,</td>
<td></td>
<td>1.4-1.5</td>
<td>1.3-1.4</td>
<td>6th 1.3-1.4</td>
</tr>
<tr>
<td>6</td>
<td>Fri, Apr 6</td>
<td>Predicate Logic</td>
<td></td>
<td>1.6-1.7</td>
<td>1.6-1.7</td>
<td>6th 1.6-1.7</td>
</tr>
<tr>
<td>7</td>
<td>Mon, Apr 9</td>
<td>Logical Inference and Proofs</td>
<td></td>
<td>1.6-1.7</td>
<td>1.6-1.7</td>
<td>6th 1.6-1.7</td>
</tr>
<tr>
<td>8</td>
<td>Wed,</td>
<td>Predicate Logic</td>
<td></td>
<td></td>
<td></td>
<td>6th 1.6-1.7</td>
</tr>
</tbody>
</table>

TA

<table>
<thead>
<tr>
<th>Office hours</th>
<th>Room</th>
</tr>
</thead>
</table>

Section

<table>
<thead>
<tr>
<th>Day/Time</th>
<th>Room</th>
</tr>
</thead>
</table>

Section Materials

<table>
<thead>
<tr>
<th>Date</th>
<th>Problems</th>
<th>Solns</th>
</tr>
</thead>
</table>

Homeworks [grading guidelines, Submission guidelines]

Exams:
- Midterm exam:
  In class, Wednesday 2 May 2018,
- Final exam:
  Monday, 4 June 2018
  The final exam has been rescheduled so that both lectures can take a common exam. The times will be 230-420 pm (the original exam time for the 1.60 section) and 430-620 pm both in Students

All Course Information @ cs.uw.edu/311
All Course Information @ cs.uw.edu/311
Why not use English?
  – Turn right here...

  – Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo buffalo

  – We saw her duck
Why not use English?

– Turn right here...
  
  Does “right” mean the direction or now?

– Buffalo buffalo Buffalo buffalo buffalo buffalo buffalo buffalo Buffalo buffalo
  
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.”

– We saw her duck
  
  Does “duck” mean the animal or crouch down?
Logic: The Language of Reasoning

Why not use English?
– Turn right here...
  Does “right” mean the direction or now?
– Buffalo buffalo Buffalo buffalo buffalo buffalo buffalo Buffalo buffalo buffalo
  buffalo Buffalo buffalo buffalo
  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.”
– We saw her duck
  Does “duck” mean the animal or crouch down?

“Language” like Java or English
– Words, sentences, paragraphs, arguments...
– Today is about words and sentences
Why Learn A New Language?

Logic, as the “language of reasoning”, will help us...

– Be more precise
– Be more concise
– Figure out what a statement means more quickly
Propositions

A *proposition* is a statement that

– has a truth value, and

– is “well-formed”
Are These Propositions?

2 + 2 = 5

- Yes

The home page renders correctly in Chrome.

- Yes

Turn in your homework on Wednesday.

- No

This statement is false.

- Yes

Akjsdf!

- No

Who are you?

- No

Every positive even integer can be written as the sum of two primes.

- Yes
Are These Propositions?

$2 + 2 = 5$
   This is a proposition. It’s okay for propositions to be false.

The home page renders correctly in Chrome.
   This is a proposition. It’s okay for propositions to be false.

Turn in your homework on Wednesday.
   This is a “command” which means it doesn’t have a truth value.

This statement is false.
   This statement does not have a truth value! (If it’s true, it’s false, and vice versa.)

Akjsdf!
   This is not a proposition because it’s gibberish.

Who are you?
   This is a question which means it doesn’t have a truth value.

Every positive even integer can be written as the sum of two primes.
   This is a proposition. We don’t know if it’s true or false, but we know it’s one of them!

Goldbach's Conjecture
A **proposition** is a statement that
- has a truth value, and
- is “well-formed”

We need a way of talking about *arbitrary* ideas...

**Propositional Variables:**
**Truth Values:**
A proposition is a statement that
  – has a truth value, and
  – is “well-formed”

We need a way of talking about arbitrary ideas...

Propositional Variables: \( p, q, r, s, \ldots \)

Truth Values:
  – \( T \) for true
  – \( F \) for false
A Proposition

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”

We’d like to understand what this proposition means.

This is where logic comes in. There are pieces that appear multiple times in the phrase (e.g., “you can get measles”).

These are called atomic propositions. Let’s list them:
A Proposition

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”

We’d like to understand what this proposition means.

This is where logic comes in. There are pieces that appear multiple times in the phrase (e.g., “you can get measles”).

These are called atomic propositions. Let’s list them:

- Measles: “You can get measles”
- Mumps: “You can get mumps”
- MMR: “You had the MMR vaccine”
“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”

Measles: “You can get measles”
Mumps: “You can get mumps”
MMR: “You had the MMR vaccine”

Now, we put these together to make the sentence:

\[
((\text{Measles} \text{ and Mumps}) \text{ if not MMR}) \text{ but } ((\text{MMR} \text{ then not (Measles or Mumps)})
\]

\[
((\text{Measles} \text{ and Mumps}) \text{ if not MMR}) \text{ and } ((\text{MMR} \text{ then not (Measles or Mumps)})
\]

This is the general idea, but now, let’s define our *formal language*.
Logical Connectives

Negation (not) \( \overline{p} \)

Conjunction (and) \( p \land q \)

Disjunction (or) \( p \lor q \)

Exclusive Or \( p \oplus q \)

Implication \( p \rightarrow q \)

Biconditional \( p \leftrightarrow q \)
### Logical Connectives

<table>
<thead>
<tr>
<th>Connective</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negation (not)</td>
<td>$\neg p$</td>
<td>“You didn’t have the MMR vaccine”</td>
</tr>
<tr>
<td>Conjunction (and)</td>
<td>$p \land q$</td>
<td>“You can get measles”</td>
</tr>
<tr>
<td>Disjunction (or)</td>
<td>$p \lor q$</td>
<td>“You can get mumps”</td>
</tr>
<tr>
<td>Exclusive Or</td>
<td>$p \oplus q$</td>
<td>“You had the MMR vaccine”</td>
</tr>
<tr>
<td>Implication</td>
<td>$p \rightarrow q$</td>
<td>“You didn’t have the MMR vaccine”</td>
</tr>
<tr>
<td>Biconditional</td>
<td>$p \leftrightarrow q$</td>
<td>“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either.”</td>
</tr>
</tbody>
</table>

```
((Measles and Mumps) if not MMR) and (if MMR then not (Measles or Mumps))
```
### Logical Connectives

<table>
<thead>
<tr>
<th>Connective</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Negation (not)</td>
<td>( \neg p )</td>
</tr>
<tr>
<td>Conjunction (and)</td>
<td>( p \land q )</td>
</tr>
<tr>
<td>Disjunction (or)</td>
<td>( p \lor q )</td>
</tr>
<tr>
<td>Exclusive Or</td>
<td>( p \oplus q )</td>
</tr>
<tr>
<td>Implication</td>
<td>( p \rightarrow q )</td>
</tr>
<tr>
<td>Biconditional</td>
<td>( p \leftrightarrow q )</td>
</tr>
</tbody>
</table>

#### Examples

- **Measles:**
  - “You can get measles”

- **Mumps:**
  - “You can get mumps”

- **MMR:**
  - “You had the MMR vaccine”

“...you can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine then you can’t get either...”

\[
((\text{Measles and Mumps}) \text{ if not MMR}) \land ((\text{if MMR then not (Measles or Mumps)})
\]

\[
((\text{Measles } \land \text{ Mumps}) \text{ if } \neg \text{MMR}) \land (\text{if MMR then } \neg (\text{Measles } \lor \text{ Mumps}))
\]
Some Truth Tables

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \oplus q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T$</td>
<td>$T$</td>
<td>$F$</td>
</tr>
<tr>
<td>$T$</td>
<td>$F$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$T$</td>
<td>$T$</td>
</tr>
<tr>
<td>$F$</td>
<td>$F$</td>
<td>$F$</td>
</tr>
</tbody>
</table>
### Some Truth Tables

<table>
<thead>
<tr>
<th>$p$</th>
<th>$\neg p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \lor q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \land q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \oplus q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Implication

“If it’s raining, then I have my umbrella”

It’s useful to think of implications as promises. That is “Did I lie?”

<table>
<thead>
<tr>
<th>p</th>
<th>q</th>
<th>p → q</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>It’s raining</th>
<th>It’s not raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have my umbrella</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>I do not have my umbrella</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>
Implication

“If it’s raining, then I have my umbrella”

It’s useful to think of implications as promises. That is “Did I lie?”

\[
\begin{array}{|c|c|c|}
\hline
p & q & p \rightarrow q \\
\hline
T & T & T \\
T & F & F \\
F & T & T \\
F & F & T \\
\hline
\end{array}
\]

<table>
<thead>
<tr>
<th></th>
<th>It’s raining</th>
<th>It’s not raining</th>
</tr>
</thead>
<tbody>
<tr>
<td>I have my umbrella</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>I do not have my umbrella</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

The only lie is when:

(a) It’s raining AND

(b) I don’t have my umbrella
Implication

“If it’s raining, then I have my umbrella”

Are these true?

\[ 2 + 2 = 4 \rightarrow \text{earth is a planet} \]

\[ 2 + 2 = 5 \rightarrow 26 \text{ is prime} \]
Implication

“If it’s raining, then I have my umbrella”

Are these true?

\[ 2 + 2 = 4 \rightarrow \text{earth is a planet} \]

The fact that these are unrelated doesn’t make the statement false! “2 + 2 = 4” is true; “earth is a planet” is true. \( T \rightarrow T \) is true. So, the statement is true.

\[ 2 + 2 = 5 \rightarrow 26 \text{ is prime} \]

Again, these statements may or may not be related. “2 + 2 = 5” is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!
(1) “I have collected all 151 Pokémon if I am a Pokémon master”
(2) “I have collected all 151 Pokémon only if I am a Pokémon master”

These sentences are implications in opposite directions:
(1) “I have collected all 151 Pokémon if I am a Pokémon master”
(2) “I have collected all 151 Pokémon only if I am a Pokémon master”

These sentences are implications in opposite directions:
(1) “Pokémon masters have all 151 Pokémon”
(2) “People who have 151 Pokémon are Pokémon masters”

So, the implications are:
(1) If I am a Pokémon master, then I have collected all 151 Pokémon.
(2) If I have collected all 151 Pokémon, then I am a Pokémon master.
$p \rightarrow q$

Implication:

- $p$ implies $q$
- whenever $p$ is true $q$ must be true
- if $p$ then $q$
- $q$ if $p$
- $p$ is sufficient for $q$
- $p$ only if $q$
- $q$ is necessary for $p$
Biconditional: $p \iff q$

- $p$ iff $q$
- $p$ is equivalent to $q$
- $p$ implies $q$ and $q$ implies $p$
- $p$ is necessary and sufficient for $q$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \iff q$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine you can’t get either.”

$$((\text{Measles} \land \text{Mumps}) \text{ if } \neg \text{MMR}) \land (\text{if MMR then } \neg (\text{Measles} \lor \text{Mumps}))$$

$$\neg \text{MMR} \rightarrow (\text{Measles} \land \text{Mumps})) \land (\text{MMR } \rightarrow \neg (\text{Measles} \lor \text{Mumps}))$$
Understanding the Vaccine Sentence

“You can get measles and mumps if you didn’t have the MMR vaccine, but if you had the MMR vaccine you can’t get either.”

\[
\begin{align*}
((\text{Measles} \land \text{Mumps}) \text{ if } \neg \text{MMR}) \land (\text{if } \text{MMR} \text{ then } \neg (\text{Measles} \lor \text{Mumps}))
\end{align*}
\]

\[
(\neg \text{MMR} \rightarrow (\text{Measles} \land \text{Mumps})) \land (\text{MMR} \rightarrow \neg (\text{Measles} \lor \text{Mumps}))
\]

Define shorthand ...

\[
\begin{align*}
p &: \text{MMR} \\
q &: \text{Measles} \\
r &: \text{Mumps}
\end{align*}
\]

\[
(\neg p \rightarrow (q \land r)) \land (p \rightarrow \neg (q \lor r))
\]
Analyzing the Vaccine Sentence with a Truth Table

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$r$</th>
<th>$\neg p$</th>
<th>$q \land r$</th>
<th>$\neg p \to (q \land r)$</th>
<th>$q \lor r$</th>
<th>$\neg (q \lor r)$</th>
<th>$p \to \neg (q \lor r)$</th>
<th>$(\neg p \to (q \land r)) \land (p \to \neg (q \lor r))$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
</tbody>
</table>
Analyzing the Vaccine Sentence with a Truth Table

<p>| | | | | | | | | | | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$p$</td>
<td>$q$</td>
<td>$r$</td>
<td>$\neg p$</td>
<td>$q \land r$</td>
<td>$\neg p \rightarrow (q \land r)$</td>
<td>$q \lor r$</td>
<td>$\neg (q \lor r)$</td>
<td>$p \rightarrow \neg (q \lor r)$</td>
<td>$(\neg p \rightarrow (q \land r)) \land (p \rightarrow \neg (q \lor r))$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>F</td>
<td>F</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Biconditional: $p \leftrightarrow q$

- $p$ iff $q$
- $p$ is equivalent to $q$
- $p$ implies $q$ and $q$ implies $p$
- $p$ is necessary and sufficient for $q$

<table>
<thead>
<tr>
<th>$p$</th>
<th>$q$</th>
<th>$p \leftrightarrow q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>