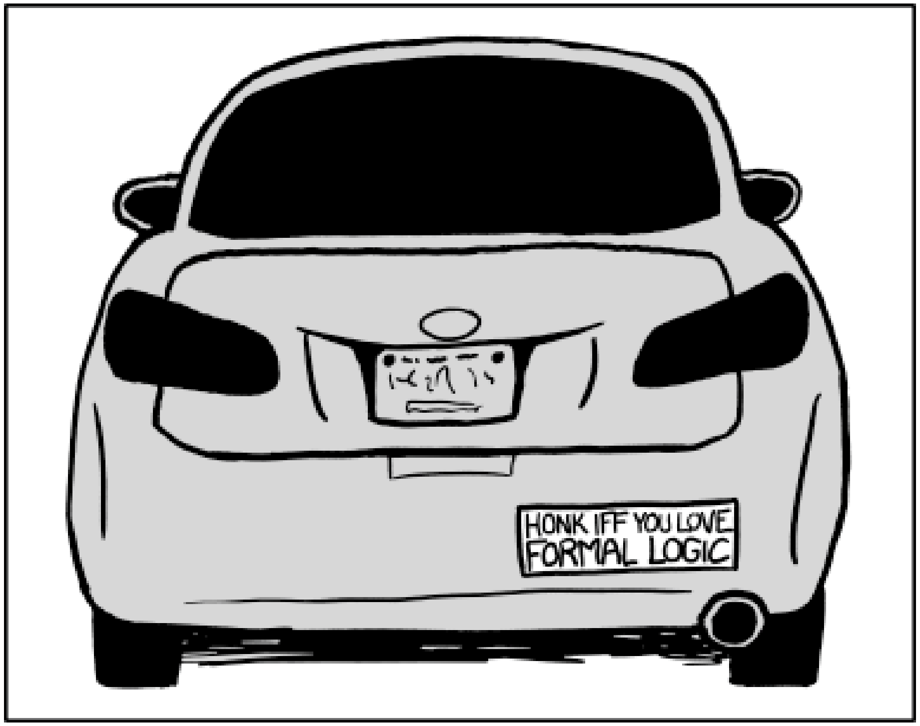
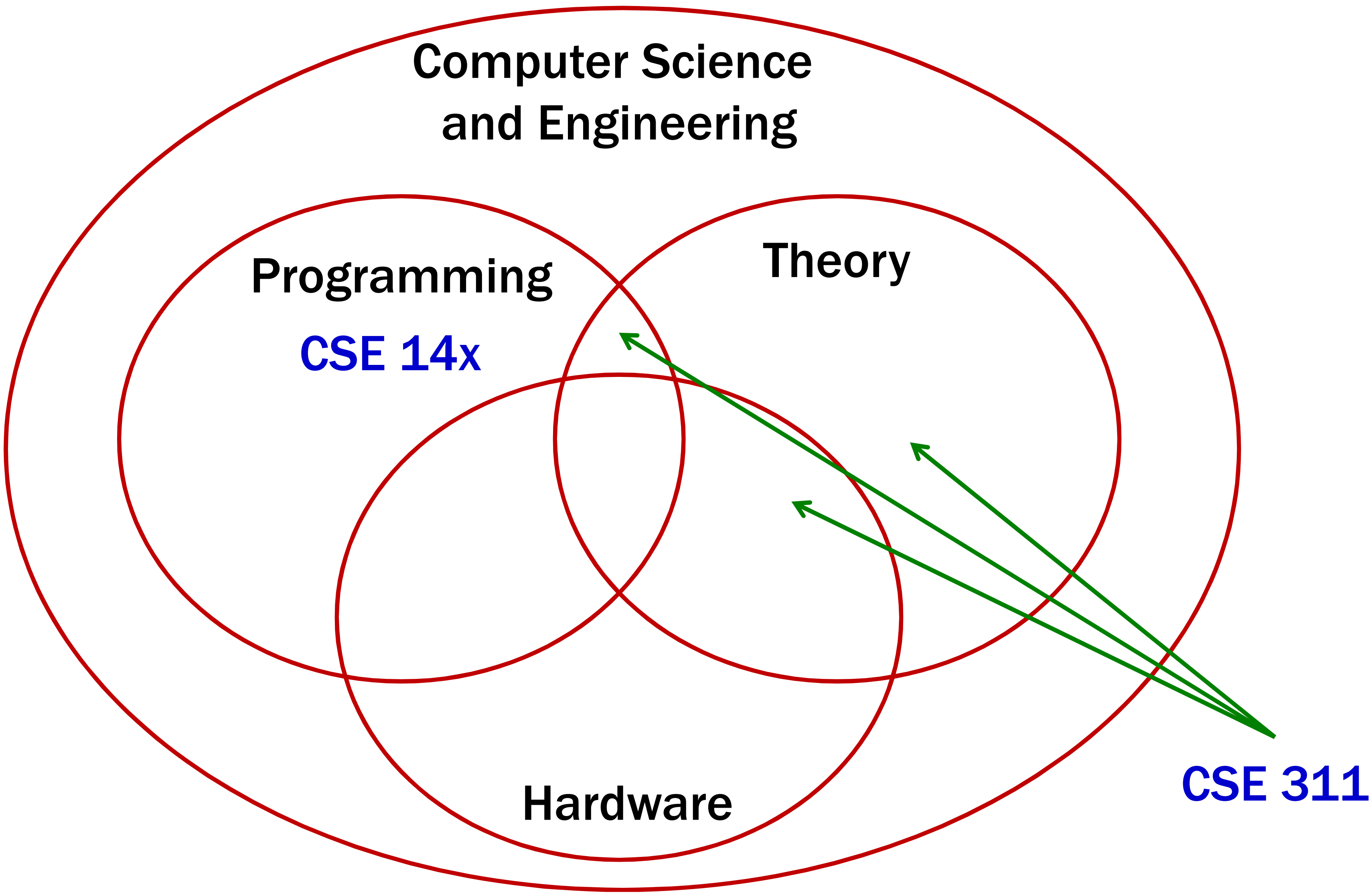
CSE 311: Foundations of Computing I

Lecture 1: Propositional Logic



Some Perspective



About the Course

We will study the *theory* needed for CSE:

Logic:

How can we describe ideas *precisely*?

Formal Proofs:

How can we be *positive* we're correct?

Number Theory:

How do we keep data *secure?*

Relations/Relational Algebra:

How do we store information?

Finite State Machines:

How do we design hardware and software?

Turing Machines:

Are there problems computers *can't* solve?

About the Course

It's about perspective!

- Tools for reasoning about difficult problems
- Tools for communicating ideas, methods, objectives…
- Tools for automating difficult problems
- Fundamental structures for computer science

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This is NOT a programming course!

Instructors

Paul Beame

Section B
MWF 10:30-11:20 in MUE 153

Office Hours:
MF 11:30-12:00 and M 3:00-4:00
CSE 668

Kevin Zatloukal

Section A
MWF 1:30-2:20 in SIG 134

Office Hours:
WF 2:30-3:00 and M 12:00-1:00
CSE 212

Office hours are for students in both sections

TAs and Administrivia

Teaching Assistants:

- Darin Chin
- Daniel Fuchs
- Yuqi Huang
- Joy Ji
- Kaiyu Zheng
- Joshua Fan
- Kush Gupta
- Sean Jaffe
- Cheng Ni

Section:

Thursdays

– starting this week

Office Hours: TBA

Homework:

Due WED at 11:59 pm online

Write up individually

Extra Credit

Grading (roughly):

50% Homework

15-20% Midterm

30-35% Final Exam

(Optional) Book:

Rosen: Readings for 6th (used) or 7th (cut down) editions.

Good for practice with solved problems

All Course Information @ cs.uw.edu/311

Administrivia

CSE 311: Foundations of Computing I

Spring, 2018

Paul Beame

> **Section A: MWF 10:30-11:20, MUE 153**
> Office hours: MF 11:30-12:00 and TBA
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Email and discussion:

email list: cse311_sp18 [archives]
Please send any e-mail about the course to **cse311-staff@cs**.

Discussion Board (moderated by TBA)
Use this board to discuss the content of the course. That includes everything **except** the solutions to current homework problems. Feel free to discuss homeworks and exams from past incarnations of the course, and any confusion over topics discussed in class. It is also acceptable to ask for *clarifications* about the statement of homework problems, but not about their solutions.

Textbook:

There is no required text for the course. Especially over the first 6-7 weeks of the course, the following textbook can be a useful companion: Rosen, *Discrete Mathematics and Its Applications*, McGraw-Hill. We will support two versions equally: (1) a special reduced version of the 7th edition which at $60 costs < 1/4 of the ridiculous price of the full text, and (2) the 6th edition, which is available used through the bookstore for even less money. It should also be available on short-term loan from the Engineering Library.

Course Calendar

Lectures

#	date	topic	slides	inked (A)	inked (B)	reading (Rosen)
1	Mon, Mar 26	Propositional Logic	pdf			1.1, 1.2 (7th) 1.1 (6th)
2	Wed, Mar 28	Logic/Gates				1.1-1.3 (7th) 1.1-1.2 (6th)
3	Fri, Mar 30	More Logic/Circuits				12.1-12.3 (7th) 11.1-11.3 (6th)
4	Mon, Apr 2	Boolean Algebra/Circuits				12.1-12.3 (7th) 11.1-11.3 (6th)
5	Wed, Apr 4	Canonical Forms, Predicate Logic				1.4-1.5 (7th) 1.3-1.4 (6th)
6	Fri, Apr 6	Predicate Logic				1.6-1.7 (7th) 1.5-1.7 (6th)
7	Mon, Apr 9	Logical Inference and Proofs				1.6-1.7 (7th) 1.5-1.7 (6th)
8	Wed,	Predicate Logic Proofs				1.6-1.7 (7th) 1.5-1.7 (6th)

TA	Office hours	Room

Section	Day/Time	Room

Section Materials	Date	Problems	Solns

Homeworks [Grading guidelines, Submission guidelines]:

Exams:

- **Midterm exam**:
  In class, Wednesday 2-May-2018,
- **Final exam**:
  Monday, 4-June-2018
  The **final exam has been rescheduled** so that both lectures can take a common exam. The times will be 2:30-4:20 pm (the original exam time for the 1:30 section) and 4:30-6:20 pm both in . **Students**

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Midterm: Wed, May 2 in class

Final Exam: Mon, Jun 4

- A section 2:30-4:20
- B section 4:30-6:20
  - Not at 8:30-10:20 time in exam schedule
- Location TBA

All Course Information @ cs.uw.edu/311

Logic: The Language of Reasoning

Why not use English?

- Turn right here…

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo

- We saw her duck

Logic: The Language of Reasoning

Why not use English?

- Turn right here…

  Does “right” mean the direction or now?

- Buffalo buffalo Buffalo buffalo buffalo buffalo Buffalo buffalo

  This means “Bison from Buffalo, that bison from Buffalo bully, themselves bully bison from Buffalo.

- We saw her duck

  Does “duck” mean the animal or crouch down?

Logic: The Language of Reasoning

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"Language" like Java or English

- Words, sentences, paragraphs, arguments…
- Today is about *words* and *sentences*

Why Learn A New Language?

Logic, as the “language of reasoning”, will help us…

- Be more precise
- Be more concise
- Figure out what a statement means more quickly

Propositions

A ***proposition*** is a statement that

- has a truth value, and
- is "well-formed"

"If I were to ask you out, would your answer to that question be the same as your answer to this one?"

Are These Propositions?

2 + 2 = 5

Y

The home page renders correctly in Chrome.

Y

Turn in your homework on Wednesday.

N

This statement is false.

N

?

Akjsdf!

N

Who are you?

N

Every positive even integer can be written as the sum of two primes.

Y

Are These Propositions?

2 + 2 = 5

This is a proposition. It's okay for propositions to be false.

The home page renders correctly in Chrome.

This is a proposition. It's okay for propositions to be false.

Turn in your homework on Wednesday.

This is a "command" which means it doesn't have a truth value.

This statement is false.

This statement does not have a truth value! (If it's true, it's false, and vice versa.)

Akjsdf!

This is not a proposition because it's gibberish.

Who are you?

This is a question which means it doesn't have a truth value.

Every positive even integer can be written as the sum of two primes.

Goldbach's conjecture

This is a proposition. We don't know if it's true or false, but we know it's one of them!

Propositions

A **proposition** is a statement that

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We need a way of talking about *arbitrary* ideas…

Propositional Variables:

Truth Values:

Propositions

A **proposition** is a statement that

- has a truth value, and
- is "well-formed"

We need a way of talking about *arbitrary* ideas…

Propositional Variables: $p, q, r, s, \ldots$

Truth Values:

- **T** for **true**
- **F** for **false**

A Proposition

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine then you can't get either."

We'd like to *understand* what this proposition means.

This is where logic comes in. There are pieces that appear multiple times in the phrase (e.g., "you can get measles").

These are called atomic propositions. Let's list them:

A Proposition

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine then you can't get either."

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These are called atomic propositions. Let's list them:

Measles: "You can get measles"

Mumps: "You can get mumps"

MMR: "You had the MMR vaccine"

Putting Them Together

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine then you can't get either."

Measles: **"You can get measles"**

Mumps: **"You can get mumps"**

MMR: **"You had the MMR vaccine"**

Now, we put these together to make the sentence:

((Measles **and** Mumps) **if not** MMR) **but** (**if** MMR **then not** (Measles **or** Mumps))

((Measles **and** Mumps) **if not** MMR) **and** (**if** MMR **then not** (Measles **or** Mumps))

This is the general idea, but now, let's define our *formal language.*

Logical Connectives

Negation (not)	$\neg p$
Conjunction (and)	$p \wedge q$
Disjunction (or)	$p \vee q$
Exclusive Or	$p \oplus q$
Implication	$p \longrightarrow q$
Biconditional	$p \leftrightarrow q$

xor

Logical Connectives

Negation (not) $\neg p$

Conjunction (and) $p \land q$

Disjunction (or) $p \lor q$

Exclusive Or $p \oplus q$

Implication $p \longrightarrow q$

Biconditional $p \leftrightarrow q$

Measles:

"You can get measles"

Mumps:

"You can get mumps"

MMR:

"You had the MMR vaccine"

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine then you can't get either."

((Measles **and** Mumps) **if not** MMR) **and** (**if** MMR **then not** (Measles **or** Mumps))

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Measles:
"You can get measles"
Mumps:
"You can get mumps"
MMR:
"You had the MMR vaccine"

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine then you can't get either."

((Measles **and** Mumps) **if not** MMR) **and** (**if** MMR **then not** (Measles **or** Mumps))

((Measles ∧ Mumps) **if** ¬MMR) ∧ (**if** MMR **then** ¬(Measles ∨ Mumps))

Some Truth Tables

p	$\neg p$
T	F
F	T

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
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T	T	T
T	F	T
F	T	T
F	F	F

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Implication

"If it's raining, then I have my umbrella"

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

It's useful to think of implications as promises. That is "Did I lie?"

	It's raining	It's not raining
I have my umbrella	NO	NO
I do not have my umbrella	YES	NO

Implication

"If it's raining, then I have my umbrella"

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

It's useful to think of implications as promises. That is "Did I lie?"

	It's raining	It's not raining
I have my umbrella	No	No
I do not have my umbrella	**Yes**	No

The only lie is when:

(a) It's raining AND

(b) I don't have my umbrella

Implication

"If it's raining, then I have my umbrella"

p	q	$p \to q$
T	T	T
T	F	F
F	T	T
F	F	T

Are these true?

$$2 + 2 = 4 \to \text{earth is a planet}$$

$$2 + 2 = 5 \to \text{26 is prime}$$

Implication

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

"If it's raining, then I have my umbrella"

Are these true?

$$2 + 2 = 4 \rightarrow \text{earth is a planet}$$

The fact that these are unrelated doesn't make the statement false! "2 + 2 = 4" is true; "earth is a planet" is true. $T \rightarrow T$ is true. So, the statement is true.

$$2 + 2 = 5 \rightarrow 26 \text{ is prime}$$

Again, these statements may or may not be related. "2 + 2 = 5" is false; so, the implication is true. (Whether 26 is prime or not is irrelevant).

Implication is not a causal relationship!

$p \rightarrow q$

(1) "I have collected all 151 Pokémon if I am a Pokémon master"

(2) "I have collected all 151 Pokémon only if I am a Pokémon master"

These sentences are implications in opposite directions:

$p \rightarrow q$

(1) "I have collected all 151 Pokémon if I am a Pokémon master"

(2) "I have collected all 151 Pokémon only if I am a Pokémon master"

These sentences are implications in opposite directions:

(1) "Pokémon masters have all 151 Pokémon"

(2) "People who have 151 Pokémon are Pokémon masters"

So, the implications are:

(1) *If* I am a Pokémon master, *then* I have collected all 151 Pokémon.

(2) *If* I have collected all 151 Pokémon, *then* I am a Pokémon master.

$p \rightarrow q$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Implication:

- p implies q
- whenever p is true q must be true
- if p then q
- q if p
- p is sufficient for q
- p only if q
- *q is necessary for p*

Biconditional: $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*
- *p* is necessary and sufficient for *q*

p	q	$p \leftrightarrow q$

Back to our Vaccine Sentence Translation…

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine you can't get either."

↓

((Measles ∧ Mumps) **if** ¬MMR) ∧ (**if** MMR **then** ¬(Measles ∨ Mumps))

↓

$$(\neg MMR \rightarrow (Measles \wedge Mumps)) \wedge (MMR \rightarrow \neg(Measles \vee Mumps))$$

Understanding the Vaccine Sentence

"You can get measles and mumps if you didn't have the MMR vaccine, but if you had the MMR vaccine you can't get either."

((Measles ∧ Mumps) **if** ¬MMR) ∧ (**if** MMR **then** ¬(Measles ∨ Mumps))

(¬MMR → (Measles ∧ Mumps)) ∧ (MMR → ¬(Measles ∨ Mumps))

Define shorthand ...

p : MMR
q : Measles
r : Mumps

$$(\neg p \rightarrow (q \wedge r)) \wedge (p \rightarrow \neg(q \vee r))$$

Analyzing the Vaccine Sentence with a Truth Table

p	q	r	$\neg p$	$q \wedge r$	$\neg p \longrightarrow (q \wedge r)$	$q \vee r$	$\neg(q \vee r)$	$p \rightarrow \neg(q \vee r)$	$\big(\neg p \longrightarrow (q \wedge r)\big) \wedge (p \rightarrow \neg(q \vee r))$
T	T	T							
T	T	F							
T	F	T							
T	F	F							
F	T	T							
F	T	F							
F	F	T							
F	F	F							

Analyzing the Vaccine Sentence with a Truth Table

p	q	r	$\neg p$	$q \wedge r$	$\neg p \longrightarrow (q \wedge r)$	$q \vee r$	$\neg(q \vee r)$	$p \rightarrow \neg(q \vee r)$	$\big(\neg p \longrightarrow (q \wedge r)\big) \wedge (p \rightarrow \neg(q \vee r))$
T	T	T	F	T	T	T	F	F	F
T	T	F	F	F	T	T	F	F	F
T	F	T	F	F	T	T	F	F	F
T	F	F	F	F	T	F	T	T	T
F	T	T	T	T	T	T	F	T	T
F	T	F	T	F	F	T	F	T	F
F	F	T	T	F	F	T	F	T	F
F	F	F	T	F	F	F	T	T	F

Biconditional: $p \leftrightarrow q$

- *p* iff *q*
- *p* is equivalent to *q*
- *p* implies *q* and *q* implies *p*
- *p* is necessary and sufficient for *q*

p	q	$p \leftrightarrow q$
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**T**