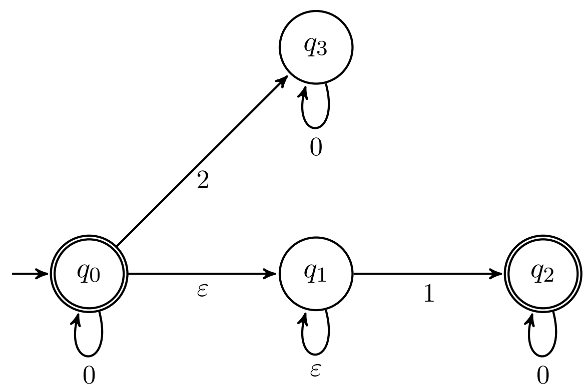
CSE 311: Foundations of Computing I

Section : Minimization, NFAs, Subset Construction, Irregularity Solutions

1. NFAs

(a) What language does the following NFA accept?



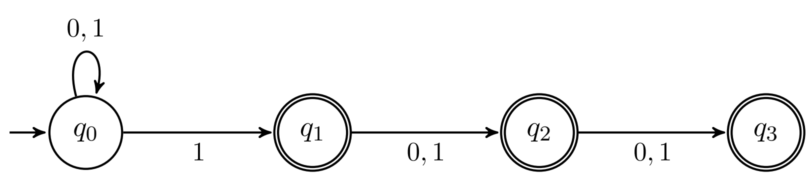
Solution:

All strings of only 0's and 1's not containing more than one 1.

(b) Create an NFA for the language "all binary strings that have a 1 as one of the last three digits".

Solution:

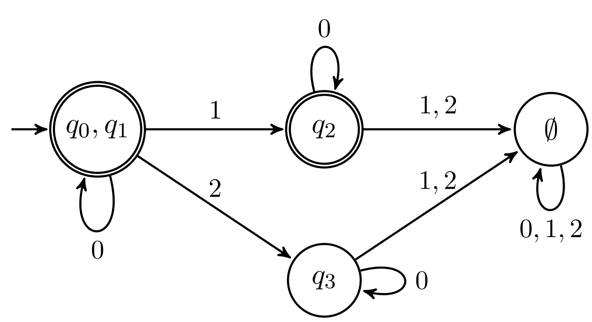
The following is one such NFA:



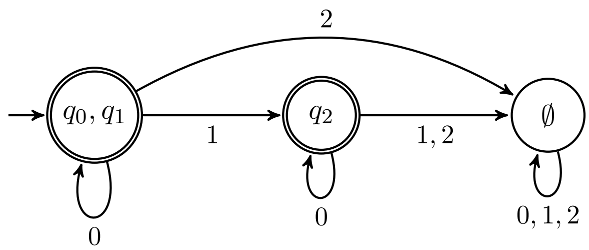
2. DFAs & Minimization

(a) Convert the NFA from 1a to a DFA, then minimize it.

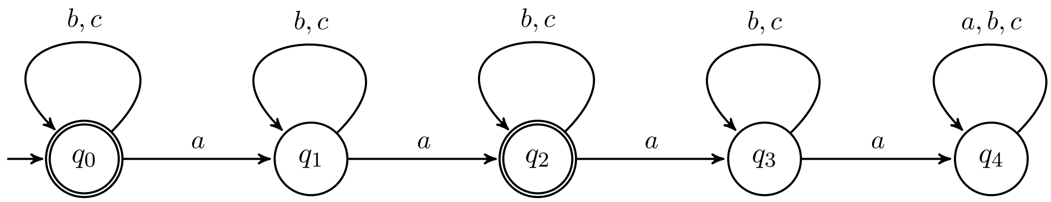
Solution:



Here is the minimized form:



(b) Minimize the following DFA:



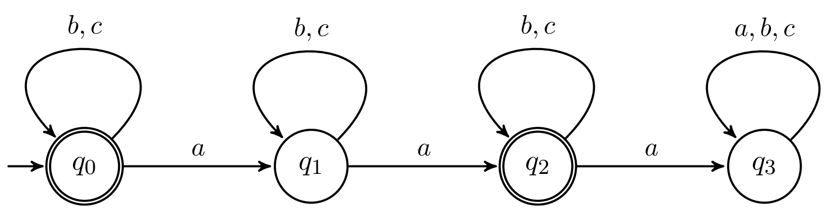
Solution:

Step 1: q_0, q_2 are final states and the rest are not final. So, we start with the initial partition with the following groups: group 1 is $\{q_0, q_2\}$ and group 2 is $\{q_1, q_3, q_4\}$.

Step 2: q_1 is sending a to group 1 while q_3, q_4 are sending a to group 2. So, we divide group 2. We get the following groups: group 1 is $\{q_0, q_2\}$, group 3 is $\{q_1\}$ and group 4 is $\{q_3, q_4\}$.

Step 3: q_0 is sending a to group 3 and q_2 is sending a to group 4. So, we divide group 1. We will have the following groups: group 3 is $\{q_1\}$, group 4 is $\{q_3, q_4\}$, group 5 is $\{q_0\}$ and group 6 is $\{q_2\}$.

The minimized DFA is the following:



3. Irregularity

(a) Let $\Sigma = \{0, 1\}$. Prove that $\{0^n 1^n 0^n \ : \ n \geq 0\}$ is not regular.

Solution:

Let $L = \{0^n 1^n 0^n \ : \ n \geq 0\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{0^n 1^n : n \geq 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are $0^i 1^i$ and $0^j 1^j$ for some $i, j \geq 0$ such that $i \neq j$. Append the string 0^i to both of these strings. The two resulting strings are:

$a = 0^i 1^i 0^i$ Note that $a \in L$.

$b = 0^j 1^j 0^i$ Note that $b \notin L$, since $i \neq j$.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.

(b) Let $\Sigma = \{0, 1, 2\}$. Prove that $\{0^n (12)^m \ : \ n \geq m \geq 0\}$ is not regular.

Solution:

Let $L = \{0^n(12)^m \ : \ n \geq m \geq 0\}$. Let D be an arbitrary DFA, and suppose for contradiction that D accepts L. Consider $S = \{0^n : n \geq 0\}$. Since S contains infinitely many strings and D has a finite number of states, two strings in S must end up in the same state. Say these strings are 0^i and 0^j for some $i, j \geq 0$ such that $i > j$. Append the string $(12)^i$ to both of these strings. The two resulting strings are:

$a = 0^i(12)^i$ Note that $a \in L$.

$b = 0^j(12)^i$ Note that $b \notin L$, since $i > j$.

Since a and b end up in the same state, but $a \in L$ and $b \notin L$, that state must be both an accept and reject state, which is a contradiction. Since D was arbitrary, there is no DFA that recognizes L, so L is not regular.