

CSE 311: Foundations of Computing I

Section : Induction and Strong Induction

1. Induction

(a) Prove that $9 \mid n^3 + (n + 1)^3 + (n + 2)^3$ for all $n > 1$ by induction.

(b) Prove that $6n + 6 < 2^n$ for all $n \geq 6$.

(c) Define

$$H_i = 1 + \frac{1}{2} + \cdots + \frac{1}{i}$$

Prove that $H_{2^n} \geq 1 + \frac{n}{2}$ for $n \in \mathbb{N}$.

2. Strong Induction

Xavier Cantelli owns some rabbits. The number of rabbits he has in any given year is described by the function f :

$$f(0) = 0$$

$$f(1) = 1$$

$$f(n) = 2f(n - 1) - f(n - 2) \text{ for } n \geq 2$$

Determine, with proof, the number, $f(n)$, of rabbits that Cantelli owns in year n .