CSE 311: Foundations of Computing I

Section : Sets and Modular Arithmetic

1. How Many Elements?

For each of these, how many elements are in the set? If the set has infinitely many elements, say .

(a) $A = \{1, 2, 3, 2\}$

(b) $B = \{\{\}, \{\{\}\}, \{\{\}, \{\}\}, \{\{\}, \{\}, \{\}\}, \ldots\}$

(c) $C = A \times (B \cup \{7\})$

(d) $D = \varnothing$

(e) $E = \{\varnothing\}$

(f) $F = \mathcal{P}(\{\varnothing\})$

2. Set = Set

Prove the following set identities.

(a) Let the universal set be $\mathcal{U}$. Prove $A \cap \overline{B} \subseteq A \setminus B$ for any sets A, B.

(b) Prove that $(A \cap B) \times C \subseteq A \times (C \cup D)$ for any sets A, B, C, D.

3. Modular Arithmetic

(a) Prove that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

(b) Prove that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

4. Casting Out Nines

Let $n \in \mathbb{N}$. Prove that if $n \equiv 0$ (mod 9), then the sum of the digits of n is a multiple of 9.

You may use without proof that for any integer $i \geq 1$, $a \equiv b$ (mod m) $\rightarrow a^i \equiv b^i$ (mod m). (This is trivial for $i = 1$ and we already proved that it holds when $i = 2$. We will later prove this for all larger i by mathematical induction.)