CSE 311: Foundations of Computing I

Section : Inference Solutions

1. Formal Proof (Direct Proof Rule)

Show that $\neg p \to s$ follows from $p \lor q$, $q \to r$ and $r \to s$.

Solution:

1.	$p \lor q$		[Given]
2.	$q \to r$		[Given]
3.	$r \to s$		[Given]
	4.1.	$\neg p$	[Assumption]
	4.2.	q	[Elim of $\lor$: 1, 4.1]
	4.3.	r	[MP of 4.2, 2]
	4.4.	s	[MP 4.3, 3]
4.	$\neg p \to s$		[Direct Proof Rule]

2. Formal Proof

Show that $\neg p$ follows from $\neg(\neg r \lor t)$, $\neg q \lor \neg s$ and $(p \to q) \land (r \to s)$.

Solution:

1.	$\neg(\neg r \lor t)$	[Given]
2.	$\neg q \lor \neg s$	[Given]
3.	$(p \to q) \land (r \to s)$	[Given]
4.	$\neg\neg r \land \neg t$	[DeMorgan's Law, 1]
5.	$\neg\neg r$	[Elim of $\land$: 4]
6.	r	[Double Negation, 5]
7.	$r \to s$	[Elim of $\land$, 3]
8.	s	[MP, 6,7]
9.	$\neg q$	[Elim of $\lor$, 2, 8]
10.	$p \to q$	[Elim of $\land$, 3]
11.	$\neg q \to \neg p$	[Contrapositive, 10]
12.	$\neg p$	[MP, 9,11]

3. Formal Proofs in Predicate Logic

For this question only, write *formal proofs*.

(a) Prove $\forall x\ (R(x) \land S(x))$ given $\forall x\ (P(x) \to (Q(x) \land S(x)))$, and $\forall x\ (P(x) \land R(x))$.

Solution:

1.	Let x be arbitrary.	
2.	$\forall x\ (P(x) \wedge R(x))$	[Given]
3.	$P(x) \wedge R(x)$	[Elim $\forall$: 2]
4.	$P(x)$	[Elim $\wedge$: 3]
5.	$R(x)$	[Elim $\wedge$: 3]
6.	$\forall x\ (P(x) \to (Q(x) \wedge S(x)))$	[Given]
7.	$P(x) \to (Q(x) \wedge S(x))$	[Elim $\forall$: 6]
8.	$Q(x) \wedge S(x)$	[MP: 4, 7]
9.	$S(x)$	[Elim $\wedge$: 8]
10.	$R(x) \wedge S(x)$	[Intro $\wedge$: 5, 9]
11.	$\forall x\ (R(x) \wedge S(x))$	[Intro $\forall$: 10]

(b) Prove $\exists x\ \neg R(x)$ given $\forall x\ (P(x) \vee Q(x))$, $\forall x\ (\neg Q(x) \vee S(x))$, $\forall x\ (R(x) \to \neg S(x))$, and $\exists x\ \neg P(x)$.

Solution:

1.	$\exists x\ \neg P(x)$	[Given]
2.	$\neg P(c)$	[Elim $\exists$: 1]
3.	$\forall x\ (P(x) \vee Q(x))$	[Given]
4.	$P(c) \vee Q(c)$	[Elim $\forall$: 3]
5.	$Q(c)$	[Elim $\vee$: 2, 4]
6.	$\forall x\ (\neg Q(x) \vee S(x))$	[Given]
7.	$\neg Q(c) \vee S(c)$	[Elim $\forall$: 6]
8.	$S(c)$	[Elim $\vee$: 5, 7]
9.	$\forall x\ (R(x) \to \neg S(x))$	[Given]
10.	$R(c) \to \neg S(c)$	[Elim $\forall$: 9]
11.	$\neg\neg S(c) \to \neg R(c)$	[Contrapositive: 10]
12.	$S(c) \to \neg R(c)$	[Double Negation: 11]
13.	$\neg R(c)$	[MP: 8, 12]
14.	$\exists x\ \neg R(x)$	[Intro $\exists$: 13]

4. Odds and Ends

Prove that for every even integer, there exists an odd integer greater than that even integer.

Solution:

Let x be an arbitrary even integer. By the definition of even, we know $x = 2y$ for some corresponding integer y. Now, we define z to be the integer $2y + 1$, which is odd by the definition of odd. By algebra, $2y + 1 > 2y$ regardless of y, so we also know $z > x$. We've now shown that there exists some integer z which is both odd and greater than x. Since x was arbitrary, our conclusion applies to all even integers.