

# CSE 311: Foundations of Computing I

---

## Section 2: Equivalences and Predicate Logic

### 1. Equivalences

Prove that each of the following pairs of propositional formulae are equivalent using propositional equivalences.

(a)  $p \leftrightarrow q$                        $(p \wedge q) \vee (\neg p \wedge \neg q)$

(b)  $\neg p \rightarrow (q \rightarrow r)$                        $q \rightarrow (p \vee r)$

### 2. Non-equivalence

Prove that the following pairs of propositional formulae are not equivalent by finding inputs they differ on.

(a)  $p \rightarrow q$                        $q \rightarrow p$

(b)  $p \rightarrow (q \wedge r)$                        $(p \rightarrow q) \wedge r$

### 3. Boolean Algebra

For each of the following parts, write the logical expression using boolean algebra operators. Then, simplify it using axioms and theorems of boolean algebra.

(a)  $\neg p \vee (\neg q \vee (p \wedge q))$

(b)  $\neg(p \vee (q \wedge p))$

### 4. Canonical Forms

Consider the boolean functions  $F(A, B, C)$  and  $G(A, B, C)$  specified by the following truth table:

$A$	$B$	$C$	$F(A, B, C)$	$G(A, B, C)$
1	1	1	1	0
1	1	0	1	1
1	0	1	0	0
1	0	0	0	0
0	1	1	1	1
0	1	0	1	0
0	0	1	0	1
0	0	0	1	0

(a) Write the DNF and CNF expressions for  $F(A, B, C)$ .

(b) Write the DNF and CNF expressions for  $G(A, B, C)$ .

## 5. Translate to Logic

Express each of these system specifications using predicate, quantifiers, and logical connectives.

- (a) Every user has access to an electronic mailbox.
- (b) The system mailbox can be accessed by everyone in the group if the file system is locked.
- (c) The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.
- (d) At least one router is functioning normally if the throughput is between 100kbps and 500 kbps and the proxy server is not in diagnostic mode.

## 6. Translate to English

Translate these system specifications into English where  $F(p)$  is "Printer  $p$  is out of service",  $B(p)$  is "Printer  $p$  is busy",  $L(j)$  is "Print job  $j$  is lost," and  $Q(j)$  is "Print job  $j$  is queued". Let the domain be all printers.

- (a)  $\exists p (F(p) \wedge B(p)) \rightarrow \exists j L(j)$
- (b)  $(\forall p B(p)) \rightarrow (\exists j Q(j))$
- (c)  $\exists j (Q(j) \wedge L(j)) \rightarrow \exists p F(p)$
- (d)  $(\forall p B(p) \wedge \forall j Q(j)) \rightarrow \exists j L(j)$