CSE 311: Foundations of Computing I

Midterm Practice Questions

Logic

(a) Show that the expression $(p \rightarrow q) \rightarrow (p \rightarrow r)$ is a contingency.

(b) Give an expression that is logically equivalent to $(p \rightarrow q) \rightarrow (p \rightarrow r)$ using the logical operators $\neg$, $\vee$, and $\wedge$ (but not $\rightarrow$).

(c) Determine whether the following compound proposition is a tautology, a contradiction, or a contingency: $((s \vee p) \wedge (s \vee \neg p)) \rightarrow ((p \rightarrow q) \rightarrow r)$.

(d) Show that the following is a tautology: $(((\neg p \vee q) \wedge (p \vee r)) \rightarrow (q \vee r))$.

Boolean Algebra

Write a boolean algebra expression equivalent to $(p \rightarrow q) \rightarrow r$ that is:

(i) A sum of products

(ii) A product of sums

Predicate Logic

(a) Using the predicates:

Likes(p, f): "Person p likes to eat the food f."

Serves(r, f): "Restaurant r serves the food f."

translate the following statements into logical expressions.

(i) Every restaurant serves a food that no one likes.

(ii) Every restaurant that serves TOFU also serves a food which RANDY does not like.

(b) Let $P(x, y)$ be the predicate "$x < y$" and let the universe for all variables be the real numbers. Express each of the following statements as predicate logic formulas using P:

(i) For every number there is a smaller one.

(ii) 7 is smaller than any other number.

(iii) 7 is between a and b. (Don't forget to handle both the possibility that b is smaller than a as well as the possibility that a is smaller than b.)

(iv) Between any two different numbers there is another number.

(v) For any two numbers, if they are different then one is less than the other.

(c) Let $V(x, y)$ be the predicate "x voted for y", let $M(x, y)$ be the predicate "x received more votes than y", and let the universe for all variables be the set of all people. Express each of the following statements as predicate logic formulas using V and M:

(i) Everybody received at least one vote.

(ii) Jane and John voted for the same person.

(iii) Ross won the election. (The winner is the person who received the most votes.)

(iv) Nobody who votes for him/herself can win the election.

(v) Everybody can vote for at most one person.

(d) Find predicates $P(x)$ and $Q(x)$ such that $\forall x(P(x) \oplus Q(x))$ is true, but $\forall x P(x) \oplus \forall x Q(x)$ is false.

Formal Proofs

(a) Use rules of inference to show that if the premises $\forall x(P(x) \rightarrow Q(x))$, $\forall x(Q(x) \rightarrow R(x))$, and $\neg R(i)$, where a is in the domain, are true, then the conclusion $\neg P(i)$ is true. (Note: You do not need to give the names for the rules of inference.)

English Proofs

(a) Prove that if n is even and m is odd, then $(n+1)(m+1)$ is even.

(b) Prove or disprove:

(i) For positive integers x, p, and q, $(x \bmod p) \bmod q = x \bmod pq$.

(ii) For positive integers x, p, and q, $(x \bmod p) \bmod q = (x \bmod q) \bmod p$.

(c) Prove that the sum of an odd number and an even number is an odd number.

Induction

(a) Prove the following for all natural numbers n by induction, $\sum_{i=0}^{n} \frac{i}{2^i} = 2 - \frac{n+2}{2^n}$.

(b) Let $T(n)$ be defined by: $T(0) = 1$, $T(n) = 2nT(n-1)$ for $n \geq 1$. Prove that for all $n \geq 0$, $T(n) = 2^n n!$.

(c) Let $x_1, x_2, \ldots, x_n$ be odd integers. Prove by induction that $x_1 x_2 \cdots x_n$ is also an odd integer.

(d) Use mathematical induction to show that 3 divides $n^3 - n$ whenever n is a non-negative integer.

Euclidean Algorithm

(a) Use Euclid's algorithm to help you solve $11x \equiv 4(\mathrm{mod} 27)$ for x.

(b) Find the multiplicative inverse of 2 modulo 9 (in other words, find a solution to the equation $2x \bmod 9 = 1$.)

(c) Which integers in $\{1, 2, \ldots, 8\}$ have multiplicative inverses modulo 9?

Sets

Prove $(A \setminus B) \cap B = \varnothing$